Fuzzy and Neutrosophic Directed Superhypergraphs: Definitions, Properties, and Real-World Applications in Decision-Making

Takaaki Fujita, Florentin Smarandache

***Abstract*—Graph theory examines mathematical structures composed of vertices and edges to model relationships and connectivity [1], [2]. Hypergraphs generalize classical graphs by permitting *hyperedges* that connect more than two vertices simultaneously [3]. Hypergraphs have been further extended to *directed hypergraphs*. Superhypergraphs build on this idea by introducing iterated powerset layers, enabling hierarchical and self-referential connections among hyperedges [4]. Superhypergraphs have likewise been extended to *directed superhypergraphs*. These graph concepts have been enriched by frameworks such as fuzzy sets [5], soft sets [6], intuitionistic fuzzy sets [7], neutrosophic sets [8], vague sets [9], hesitant fuzzy sets [10], and plithogenic sets [11]. In this paper, we introduce and explore the concepts of *Fuzzy Directed Superhypergraphs* and *Neutrosophic Directed Superhypergraphs*. To enhance understanding and support future applications, we also provide concrete examples involving real-world decision-making scenarios. We expect that these contributions will stimulate further research in the areas of superhypergraphs, hypergraphs, fuzzy graphs, neutrosophic graphs, and other related mathematical frameworks.**

***Index Terms*—Superhypergraph, Hypergraph, Fuzzy Graph, Neutrosophic Graph, Fuzzy HyperGraph. Neutrosophic HyperGraph, Directed Superhypergraph**

I. Introduction

A. Graph, Hypergraph, and Superhypergraph

Network models are often formulated using graphs, where entities are represented by vertices and pairwise relations by edges [1]. However, graphs can be ill-suited for expressing relationships that simultaneously involve multiple concepts. To address this limitation, hypergraphs were introduced. A hypergraph generalizes a graph by permitting each edge to join any nonempty subset of the vertex set, thereby modeling higher-order interactions [12].

Nevertheless, hypergraphs may become inconvenient when one needs to represent concepts with more strongly hierarchical, multilayered structure. Motivated by this, a *SuperHyperGraph* is defined via iterative powerset constructions, which allow one to encode nested and hierarchical connectivity patterns [4]; this direction has attracted considerable recent interest [13], [14].

Graphs and hypergraphs offer intuitive visual metaphors for complex systems and underpin a wide range of applications in artificial intelligence, network science, data mining, informatics, chemistry, physics, and beyond [2]. By accommodating multi-level relationships, SuperHyperGraphs provide a robust framework for modeling and analyzing the intricate structures encountered in real-world networks.

Table I summarizes the key distinctions among graphs, hypergraphs, and superhypergraphs. Throughout this paper, n is taken to be a natural number unless stated otherwise.

B. Directed, Bidirected, and Multidirected Graphs

As discussed above, graphs are widely applied across numerous domains. However, when modeling concepts that inherently involve directional information, the use of *directed graphs* becomes essential[15], [16]. These structures have been further extended to *directed hypergraphs*[17] and *directed superhypergraphs* [18], [19], which have attracted growing research interest.

Related notions include *bidirected graphs*[20], [21], [22], [23], where each edge is assigned a direction at each endpoint, and *multidirected graphs* [24], which allow for multiple directed edges between vertices, possibly with different directions. These generalizations are useful for representing more nuanced or complex directional relationships in real-world systems.

C. Uncertain Graphs: Fuzzy and Neutrosophic Variants

Uncertainty is inherent in many real-world systems and must be explicitly represented. To address this, various set-based frameworks—such as fuzzy sets [25], [26], [5], intuitionistic fuzzy sets [7], neutrosophic sets [27], [28], [8], picture fuzzy sets [29], [30], soft sets [6], [31], hypersoft sets [32], rough sets [33], [34], uncertain sets [35], [36], and plithogenic sets [37]—have been introduced. These paradigms have subsequently been adapted to graph theory, yielding network models that incorporate uncertainty. Representative instances include fuzzy graphs [38], intuitionistic fuzzy graphs [39], bipolar fuzzy graphs [40], [41], bipolar fuzzy influence graphs [42], vague graphs [43], neutrosophic graphs [44], [45], plithogenic graphs [46], [47], soft graphs [48], hyper-soft graphs [49], and rough graphs [50].

In this work, we focus on the two most fundamental types: fuzzy graphs and neutrosophic graphs. In a fuzzy graph, each vertex and edge is assigned a single membership value in $[0, 1]$, quantifying the strength of connections [38]. A neutrosophic graph, by contrast, assigns three independent degrees—truth T, indeterminacy I, and falsity F—to each element, allowing the model to capture inconsistent or partially unknown relationships [44].

Table II compares the essential characteristics of fuzzy and neutrosophic graph models.

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TABLE I: Key distinctions among graph, hypergraph, and superhypergraph

Concept	Notation	Edge Type	Extension Mechanism
Graph	$G = (V, E)$	$E \subseteq \{\{u, v\} \mid u, v \in V,\ u \neq v\}$	Standard edges connect exactly two vertices.
Hypergraph	$H = (V, E)$	$E \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$	Hyperedges may join any nonempty subset of vertices.
Superhypergraph	$\mathrm{SHG}^{(n)} = (V_0, V, E)$	$V, E \subseteq \mathcal{P}^n(V_0)$	Applies an n-fold powerset to capture nested structure.

TABLE II: Comparison of Fuzzy and Neutrosophic Graph Models

Graph Type	Membership / Functions	Key Features
Fuzzy Graph	$\mu : V \times V \to [0, 1]$	Single membership value per edge, with $\mu(u, v) \leq \min\{\sigma(u), \sigma(v)\}$.
Neutrosophic Graph	$T, I, F : V \times V \to [0, 1]$	Three independent degrees (T, I, F) satisfying $0 \leq T + I + F \leq 3$.

D. Uncertain HyperGraphs: Fuzzy and Neutrosophic Variants

Each of above graph models—fuzzy and neutrosophic—can be lifted to the hypergraph setting, giving rise to fuzzy hypergraphs [51], [52] and neutrosophic hypergraphs [53], [54]. These ideas can be pushed further into "recursive nested" architectures, producing fuzzy superhypergraphs and neutrosophic superhypergraphs (see, e.g.,[4], [55]). By capturing increasingly intricate and uncertain network structures, these generalized frameworks play a vital role not only in graph theory but also across computational science, informatics, network theory, and applied mathematics[56], [57], [14], [58]. Table III presents an overview of uncertain hypergraph and superhypergraph models.

Also, for reference, we present in Table IV a concise comparison of NG, NHG, and NnSHG (single-valued, undirected).

E. Uncertain Directed Graphs: Fuzzy and Neutrosophic Variants

Fuzzy graphs and neutrosophic graphs have been extended to their directed counterparts, known as fuzzy directed graphs[59], [60] and neutrosophic directed graphs[61], [62], respectively. These structures have been widely investigated due to their applicability in modeling systems where directional relationships are essential.

Similarly, fuzzy hypergraphs and neutrosophic hypergraphs have also been generalized to include directionality, resulting in frameworks such as fuzzy directed hypergraphs [63], [64], [65], [66] and single-valued neutrosophic directed hypergraphs [67], [68], [69].

These directed models are particularly well-suited for capturing complex, real-world phenomena that inherently involve directional interactions or asymmetric relationships.

F. Our Contribution

As highlighted above, research on graphs, hypergraphs, superhypergraphs, and their directed and uncertain variants—such as fuzzy graphs and neutrosophic graphs—is of great significance. However, to the best of our knowledge, no formal study has yet been conducted on the concepts of *Fuzzy Directed SuperhyperGraphs* and *Neutrosophic Directed SuperhyperGraphs*.

In this paper, as summarized in Table V, we introduce formal definitions for these two novel structures. We explore their mathematical properties and demonstrate their applicability through concrete examples in fields such as decision-making.

By establishing these new models, we aim to provide a framework capable of intuitively representing real-world systems that involve complex, hierarchical, and directionally dependent relationships. These contributions may pave the way for more expressive and structured modeling of intricate networks and conceptual systems in various domains.

G. Structure of this Paper

This section outlines the organization of the present paper. Section 2 provides the necessary preliminaries, including overviews and definitions of key concepts such as hypergraphs, superhypergraphs, directed hypergraphs, directed superhypergraphs, fuzzy directed hypergraphs, and neutrosophic hypergraphs. Section 3 focuses on the mathematical properties and illustrative applications of fuzzy directed n-superhypergraphs. Section 4 examines the mathematical properties and practical examples of neutrosophic directed n-superhypergraphs. In Section 5, we provide a brief qualitative comparison between the concept introduced in this paper and those proposed in existing studies. Finally, Section 6 presents the conclusions of this study and discusses directions for future research.

II. Preliminaries

This section presents the fundamental concepts and definitions that underpin the discussions in this paper. Unless otherwise noted, all graphs considered here are *undirected*, *finite*, and *simple*. For detailed treatment of specific operations and related notions, the reader is referred to the appropriate literature.

A. SuperHyperGraph

In traditional graph theory, a *hypergraph* broadens the notion of a standard graph by allowing *hyperedges* that join more than two vertices. This capability makes hypergraphs particularly well suited for capturing complex, multi-way interactions in diverse applications [70], [3], [71], [72], [12]. A *SuperHyperGraph* further enhances this structure by incorporating iterated powerset operations into the hypergraph

TABLE III: Overview of Uncertain Hypergraph and Superhypergraph Models

Model	Description
Fuzzy Hypergraph	Each vertex and hyperedge is assigned a fuzzy membership value between zero and one, representing uncertain multiway connections.
Neutrosophic Hypergraph	Vertices and hyperedges carry independent truth, indeterminacy, and falsity degrees, capturing inconsistent or partial information.
Fuzzy Superhypergraph	Extends fuzzy hypergraphs via recursive powerset levels, enabling nested layers of fuzzy uncertainty in hierarchical networks.
Neutrosophic Superhypergraph	Integrates neutrosophic degrees with iterative hypergraph construction to model multi-level uncertainty across nested contexts.

TABLE IV: Concise comparison of NG, NHG, and NnSHG (single-valued, undirected).

Aspect	Neutrosophic Graph (NG)	Neutrosophic Hypergraph (NHG)	Neutrosophic n-SuperHyperGraph (NnSHG)
Underlying structure	$G = (V, E)$, $E \subseteq \binom{V}{2}$.	$H = (V, E)$, $E \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$.	Base V_0; $V \subseteq \mathcal{P}^n(V_0)$, $E \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$.
Objects	Vertices $v \in V$, edges $\{u, v\}$.	Vertices $v \in V$, hyperedges $e \subseteq V$.	n-supervertices $v \in \mathcal{P}^n(V_0)$, superedges $e \subseteq V$.
Neutrosophic degrees	$T_V, I_V, F_V : V \to [0,1]$, $T_E, I_E, F_E : E \to [0,1]$.	Same (on vertices and hyperedges).	Same (on n-supervertices and superedges).
Constraints (typical)	$0 \le T + I + F \le 3$ on vertices/edges; $T_E(e) \le \min_{x \in e} T_V(x)$ (and similarly I, F).	Same with e a hyperedge.	Same with e a superedge among n-supervertices.
Expressiveness (informal)	Uncertain *pairwise* relations.	Uncertain *multiway* relations.	Uncertain *multiway* relations among *nested/hierarchical* entities.
Special case	—	NG is NHG with $\lvert e \rvert = 2$.	N0SHG is NHG on V_0.

TABLE V: Overview of Directed Uncertain Graph Models

Model	Description	Introduced
Fuzzy Directed Graph	Each vertex and directed edge has a fuzzy membership value between zero and one to capture uncertain directionality.	Prior work
Neutrosophic Directed Graph	Each vertex and directed edge is assigned three independent truth, indeterminacy, and falsity degrees to model partial uncertainty.	Prior work
Fuzzy Directed HyperGraph	Extends fuzzy directed graphs by permitting directed hyperedges with fuzzy membership for uncertain multiway relations.	Prior work
Neutrosophic Directed HyperGraph	Extends neutrosophic directed graphs by using directed hyperedges with neutrosophic degrees for complex relationships.	Prior work
Fuzzy Directed SuperhyperGraph	Recursive multi-level fuzzy hyperstructure with directed superedges for nested network uncertainty.	Proposed here
Neutrosophic Directed SuperhyperGraph	Recursive multi-level neutrosophic hyperstructure with directed superedges for hierarchical uncertainty modeling.	Proposed here

framework, a topic that has recently attracted significant attention [73], [58], [74], [14].

Besides its theoretical significance, SuperHyperGraphs have been employed in areas such as molecular modeling, network analysis, and signal processing [75], [76], [57]. Throughout this work, the parameter n in both the n-th powerset and the n-SuperHyperGraph is assumed to be a non-negative integer.

Definition 1 (Base Set)**.** A *base set* S serves as the origin for more elaborate constructions like powersets and hyperstructures. Formally,

$$S = \{\, x \mid x \text{ belongs to the domain of interest} \}.$$

Every element of $\mathcal{P}(S)$ or $\mathcal{P}_n(S)$ is drawn from S.

Definition 2 (Powerset)**.** The *powerset* of S, denoted $\mathcal{P}(S)$, comprises all subsets of S, including $\varnothing$ and S itself:

$$\mathcal{P}(S) = \{\, A \mid A \subseteq S \}.$$

Definition 3 (Hypergraph)**.** [77], [3] A *hypergraph* $H = (V(H), E(H))$ consists of:

- A nonempty vertex set $V(H)$.
- A collection of hyperedges $E(H)$, each of which is a nonempty subset of $V(H)$, allowing connections among multiple vertices.

Here, we assume $V(H)$ and $E(H)$ are finite.

Definition 4 (n-th Powerset)**.** [78], [79], [80], [81] The n-th powerset of H, written $\mathcal{P}_n(H)$, is defined by iteration:

$$\mathcal{P}_1(H) = \mathcal{P}(H),$$

$$\mathcal{P}_{k+1}(H) = \mathcal{P}\big(\mathcal{P}_k(H)\big) \quad (k \ge 1).$$

Likewise, the nonempty version $\mathcal{P}^*_n(H)$ satisfies

$$\mathcal{P}^*_1(H) = \mathcal{P}^*(H), \quad \mathcal{P}^*_{k+1}(H) = \mathcal{P}^*\big(\mathcal{P}^*_k(H)\big),$$

where $\mathcal{P}^*(H)$ omits the empty set.

Example 5 (Meal Planning via the 2nd and 3rd Powerset)**.** Let

$$H = \{\text{Tomato, Cheese, Basil}\}$$

be a set of basic ingredients.

$$\mathcal{P}_1(H) = \big\{\varnothing,\ \{\text{Tomato}\}, \{\text{Cheese}\}, \{\text{Basil}\},$$
$$\{\text{Tomato, Cheese}\}, \{\text{Tomato, Basil}\}, \{\text{Cheese, Basil}\},$$
$$\{\text{Tomato, Cheese, Basil}\}\big\}.$$

We interpret each nonempty subset as a simple "recipe," for instance:

$$R_1 = \{\text{Tomato, Basil}\} \quad (\text{Bruschetta}),$$
$$R_2 = \{\text{Tomato, Cheese}\} \quad (\text{Caprese Salad}).$$

$$\mathcal{P}_2(H) = \mathcal{P}\big(\mathcal{P}_1(H)\big) = \{\, M \mid M \subseteq \mathcal{P}_1(H)\}.$$

For example, a two-course menu might be

$$M_1 = \{R_1,\ R_2\} \ \in\ \mathcal{P}_2(H).$$

$$\mathcal{P}_3(H) = \mathcal{P}\big(\mathcal{P}_2(H)\big) = \{\, \mathcal{M} \mid \mathcal{M} \subseteq \mathcal{P}_2(H)\}.$$

A week-long plan could combine several menus:

$$\mathcal{M}_1 = \{M_1,\ M_2,\ M_3\} \ \in\ \mathcal{P}_3(H),$$

where each M_i is a menu drawn from $\mathcal{P}_2(H)$.

Thus, by iterating the powerset operation, one moves from individual ingredients ($\mathcal{P}_0$) to recipes ($\mathcal{P}_1$), to menus ($\mathcal{P}_2$), and finally to multi-day meal plans ($\mathcal{P}_3$), illustrating a concrete use of the n-th powerset in daily life. For reference, an overview diagram is provided in Figure 1.

Definition 6 (n-SuperHyperGraph)**.** [82], [4] Let V_0 be a finite set of base vertices. For each $k \geq 0$,

$$\mathcal{P}^0(V_0) = V_0, \quad \mathcal{P}^{k+1}(V_0) = \mathcal{P}\big(\mathcal{P}^k(V_0)\big).$$

An *n-SuperHyperGraph* is the pair

$$\text{SHG}^{(n)} = (V, E),$$
$$V \subseteq \mathcal{P}^n(V_0),\ E \subseteq \mathcal{P}^n(V_0) (E \subseteq \mathcal{P}(V)).$$

Elements of V are called *n-supervertices*, and those of E are *n-superedges*.

Example 7 (2-SuperHyperGraph on $V_0 = \{1, 2, 3\}$)**.** Set

$$V_0 = \{1, 2, 3\}.$$

Then

$$\mathcal{P}^1(V_0) = \{\varnothing, \{1\}, \{2\}, \{3\}, \{1, 2\},$$
$$\{1, 3\}, \{2, 3\}, \{1, 2, 3\}\},$$
$$\mathcal{P}^2(V_0) = \mathcal{P}\big(\mathcal{P}^1(V_0)\big) = \{\, X \mid X \subseteq \mathcal{P}^1(V_0)\}.$$

Choose three 2-supervertices:

$$v_1 = \{\{1\}, \{1, 2\}\},$$
$$v_2 = \{\{2\}, \{2, 3\}\},$$
$$v_3 = \{\{3\}, \{1, 2, 3\}\}.$$

Thus $V = \{v_1, v_2, v_3\}$. Form two 2-superedges:

$$e_1 = \{v_1, v_2\}, \quad e_2 = \{v_2, v_3\},$$

so $E = \{e_1, e_2\}$. Hence

$$\text{SHG}^{(2)} = (V, E)$$

is a valid 2-SuperHyperGraph on V_0.

For reference, Figure 2 provides an overview of this SuperHyperGraph.

B. Fuzzy n-Superhypergraph

A *fuzzy set* assigns to each element of a universe a membership degree in the interval $[0, 1]$ [5], [83], [84]. A *fuzzy graph* extends this concept by equipping both vertices and edges of a graph with membership degrees [85], [38]. A *fuzzy hypergraph* further generalizes fuzzy graphs by allowing membership degrees on vertices and hyperedges[51], [86]. A *fuzzy n-superhypergraph* extends the idea to n-superhypergraphs, assigning membership degrees to n-supervertices and n-superedges. The formal definition of a fuzzy n-superhypergraph is given as follows (cf.[58], [4]).

Definition 8 (Fuzzy n-Superhypergraph)**.** (cf.[58], [4]) Let $\text{SHG}^{(n)} = (V, E)$ be an n-Superhypergraph. A *fuzzy n-Superhypergraph* is a quadruple

$$\big(V,\ E,\ \sigma,\ \mu\big),$$

where

- $\sigma : V \to [0, 1]$ assigns to each n-supervertex v a membership degree $\sigma(v)$.
- $\mu : E \to [0, 1]$ assigns to each n-superedge e a membership degree $\mu(e)$.

These functions satisfy the *appurtenance constraint*

$$\mu(e) \ \leq\ \min_{v \in e} \sigma(v), \quad \forall\, e \in E.$$

Example 9 (Fuzzy 2-Superhypergraph on $V_0 = \{a, b\}$)**.** Let the base set be

$$V_0 = \{a, b\}.$$

Then

$$\mathcal{P}^1(V_0) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\},$$
$$\mathcal{P}^2(V_0) = \mathcal{P}\big(\mathcal{P}^1(V_0)\big).$$

From $\mathcal{P}^2(V_0)$, select two 2–supervertices:

$$v_1 = \big\{\{a\}, \{a, b\}\big\}, \quad v_2 = \{\{b\}\}.$$

Thus

$$V = \{v_1, v_2\}.$$

Choose two 2–superedges:

$$e_1 = \{\, v_1, v_2\}, \quad e_2 = \{\, v_1\}.$$

Hence

$$E = \{e_1, e_2\}.$$

Define the vertex–membership function $\sigma : V \to [0, 1]$ and the edge–membership function $\mu : E \to [0, 1]$ by

$$\sigma(v_1) = 0.7, \quad \sigma(v_2) = 0.5,$$
$$\mu(e_1) = 0.5, \quad \mu(e_2) = 0.6.$$

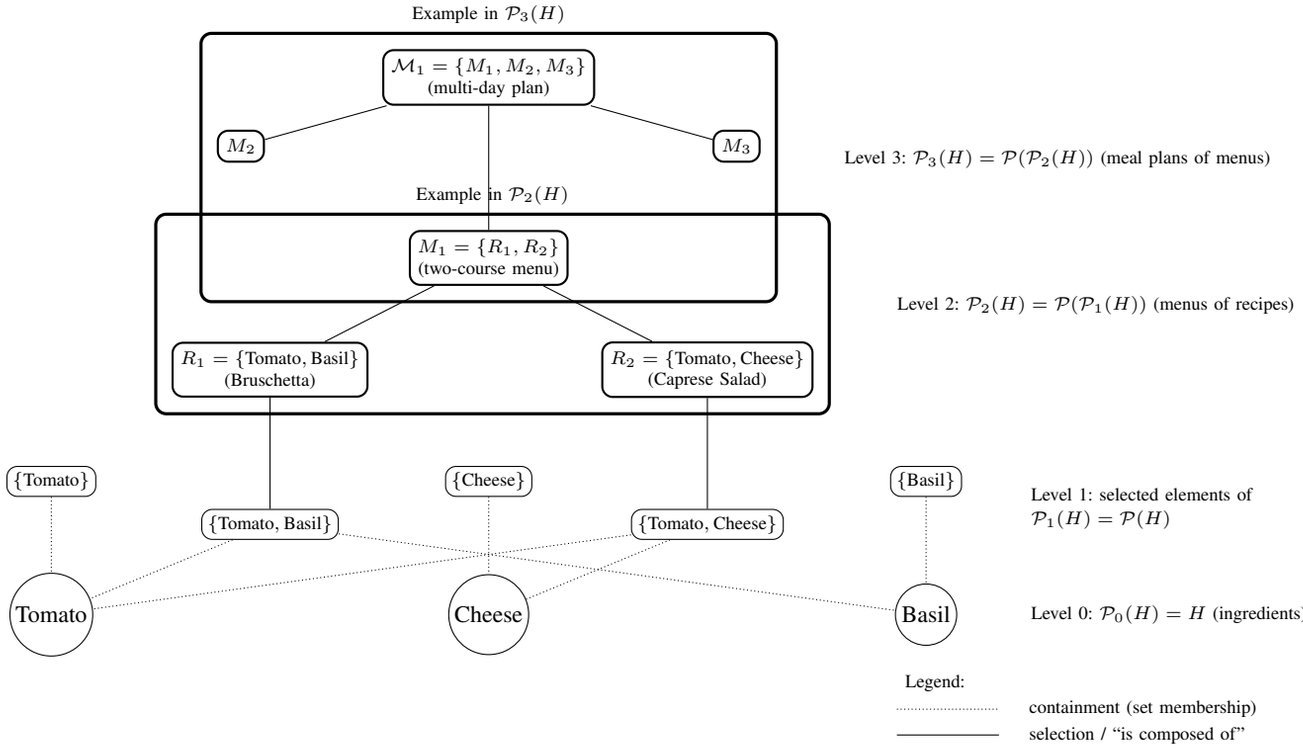

Fig. 1: A schematic diagram of the iterated powerset construction in the meal-planning example: ingredients ($\mathcal{P}_0$) $\rightarrow$ ingredient-sets/recipes ($\mathcal{P}_1$) $\rightarrow$ menus ($\mathcal{P}_2$) $\rightarrow$ meal plans ($\mathcal{P}_3$).

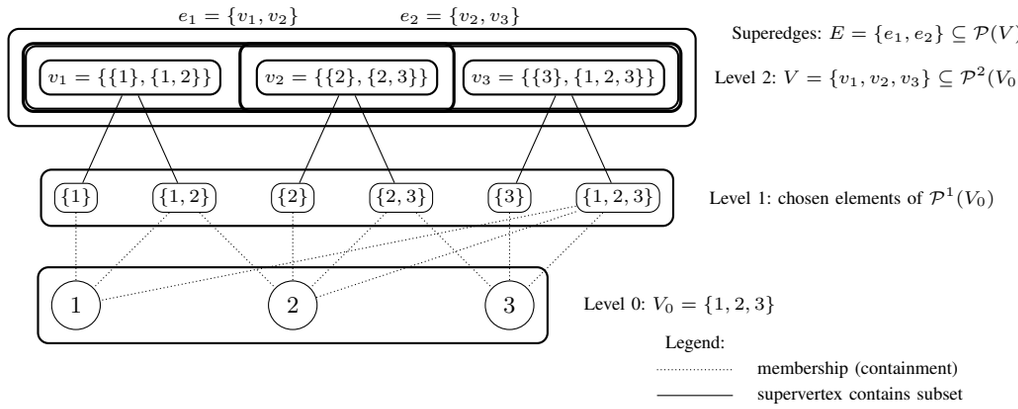

Fig. 2: A schematic visualization of the 2-SuperHyperGraph in the example ($V_0 = \{1, 2, 3\}$).

We verify the appurtenance constraint $\mu(e) \leq \min_{v \in e} \sigma(v)$:

$$\mu(e_1) = 0.5 \ \leq \ \min\{\sigma(v_1), \sigma(v_2)\} = \min\{0.7, 0.5\} = 0.5,$$

$$\mu(e_2) = 0.6 \ \leq \ \min\{\sigma(v_1)\} = 0.7.$$

Therefore $\big(V, E, \sigma, \mu\big)$ is a valid fuzzy 2–Superhypergraph.

Example 10 (Fuzzy 2-Superhypergraph for Protein Complex Modeling)**.** Let

$$V_0 = \{\, P_1,\, P_2,\, P_3,\, P_4\}$$

be a set of proteins observed in a cell.

Define three first-level complexes (1-supervertices):

$$C_1 = \{P_1,\, P_2\}, \quad C_2 = \{P_2,\, P_3\}, \quad C_3 = \{P_3,\, P_4\}.$$

These form the vertex set of the underlying hypergraph:

$$V^{(1)} = \{\, C_1,\, C_2,\, C_3\}.$$

Now consider two second-level supervertices:

$$v_1 = \{\, C_1,\, C_2\}, \quad v_2 = \{\, C_2,\, C_3\}.$$

Thus the 2-supervertex set is

$$V = \{\, v_1,\, v_2\}.$$

Form one 2-superedge linking these meta-complexes:

$$e = \{\, v_1,\, v_2\}.$$

Hence the 2-superedge set is

$$E = \{\, e\}.$$

In experiments, each meta-complex and their relation may form with varying confidence scores in [0,1]:

$$\sigma(v_1) = 0.ninety, \quad \sigma(v_2) = 0.eighty.$$

The observed interaction between v_1 and v_2 is measured as:

$$\mu(e) = 0.eighty.$$

These satisfy the appurtenance constraint

$$\mu(e) \ \leq \ \min\{\sigma(v_1),\, \sigma(v_2)\}.$$

Therefore, the fuzzy 2-superhypergraph is the quadruple

$$\big(V,\, E,\, \sigma,\, \mu\big) \ = \ \Big(\{v_1, v_2\},\, \{e\},\, \sigma, \mu\Big),$$

which captures both the hierarchical organization of protein complexes and the uncertainty in their assembly into higher-order structures.

C. Single-valued Neutrosophic n-Superhypergraph

A *Single-valued Neutrosophic n-Superhypergraph* is a concept that generalizes both the Single-valued Neutrosophic graph [87], [88], [89] and the Single-valued Neutrosophic hypergraph[90], [91]. It also extends the notion of a Fuzzy n-Superhypergraph. The formal definition and a representative example are given below(cf.[76]).

Definition 11 (Neutrosophic n-Superhypergraph)**.** (cf.[76]) Let V_0 be a finite *base set* of vertices, and for each integer $k \geq 0$ define

$$\mathcal{P}^0(V_0) = V_0,$$
$$\mathcal{P}^{k+1}(V_0) = \mathcal{P}\big(\mathcal{P}^k(V_0)\big),$$

where $\mathcal{P}(\cdot)$ denotes the usual powerset. An *n-Superhypergraph* is a pair

$$\mathrm{SHG}^{(n)} = (V,\, E), \quad V \ \subseteq \ \mathcal{P}^n(V_0), \ \ E \ \subseteq \ \mathcal{P}^n(V_0).$$

A *Neutrosophic n-Superhypergraph* is then the tuple

$$\big(V,\, E,\, T_V,\, I_V,\, F_V,\, T_E,\, I_E,\, F_E\big),$$

where

- $T_V,\, I_V,\, F_V \ : \ V \ \to \ [0,1]$ assign to each n-supervertex $v \in V$ its truth-, indeterminacy-, and falsity-membership degrees, respectively, subject to

$$0 \ \leq \ T_V(v) + I_V(v) + F_V(v) \ \leq \ 3,$$
$$\forall v \in V.$$

- $T_E,\, I_E,\, F_E \ : \ E \times V \ \to \ [0,1]$ assign to each n-superedge $e \ \in \ E$ and vertex $v \ \in \ e$ its truth-, indeterminacy-, and falsity-membership degrees, respectively, subject to

$$0 \ \leq \ T_E(e,v) + I_E(e,v) + F_E(e,v) \ \leq \ 3,$$
$$\forall e \in E, \ \forall v \in e.$$

These functions satisfy the *edge-appurtenance constraints*:

$$T_E(e,v) \ \leq \ T_V(v),$$
$$I_E(e,v) \ \leq \ I_V(v),$$
$$F_E(e,v) \ \leq \ F_V(v),$$
$$\forall\, e \in E, \ \forall\, v \in e.$$

Example 12 (Single-valued Neutrosophic 2-Superhypergraph on $V_0 = \{a, b\}$)**.** Let the base set be

$$V_0 = \{a, b\}, \quad \mathcal{P}^1(V_0) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\},$$
$$\mathcal{P}^2(V_0) = \mathcal{P}\big(\mathcal{P}^1(V_0)\big).$$

From $\mathcal{P}^2(V_0)$, select two 2-supervertices:

$$v_1 = \{\{a\}, \{a, b\}\}, \quad v_2 = \{\{b\}\}.$$

Thus

$$V = \{v_1, v_2\}.$$

Next, choose two 2-superedges (subsets of V):

$$e_1 = \{v_1, v_2\}, \quad e_2 = \{v_2\},$$

so

$$E = \{e_1, e_2\}.$$

Assign to each $v \in V$ its truth, indeterminacy, and falsity degrees:

$$T_V(v_1) = 0.8, \ I_V(v_1) = 0.1, \ F_V(v_1) = 0.1,$$
$$T_V(v_2) = 0.6, \ I_V(v_2) = 0.2, \ F_V(v_2) = 0.1.$$

For each edge $e \in E$ and each $v \in e$, define:

$$T_E(e_1, v_1) = 0.7,$$
$$I_E(e_1, v_1) = 0.05,$$
$$F_E(e_1, v_1) = 0.05,$$
$$T_E(e_1, v_2) = 0.5,$$
$$I_E(e_1, v_2) = 0.10,$$
$$F_E(e_1, v_2) = 0.05,$$
$$T_E(e_2, v_2) = 0.6,$$
$$I_E(e_2, v_2) = 0.20,$$
$$F_E(e_2, v_2) = 0.10.$$

One checks easily that for all $v \in V$,

$$0 \leq T_V(v) + I_V(v) + F_V(v) \leq 3,$$

and for all $e \in E$, $v \in e$,

$$T_E(e,v) \leq T_V(v), \quad I_E(e,v) \leq I_V(v), \quad F_E(e,v) \leq F_V(v),$$

thus verifying that

$$\big(V,\, E,\, T_V,\, I_V,\, F_V,\, T_E,\, I_E,\, F_E\big)$$

is a valid single-valued neutrosophic 2-Superhypergraph.

Example 13 (Collaborative Research Teams as a Single-valued Neutrosophic 2-Superhypergraph)**.** Let

$$V_0 = \{\text{Alice, Bob, Carol, Dave}\}$$

be a set of researchers. Then

$$\mathcal{P}^1(V_0) = \big\{\, \emptyset,$$
$$\{\text{Alice}\}, \{\text{Bob}\}, \{\text{Carol}\}, \{\text{Dave}\}, \{\text{Alice, Bob}\}, \ldots,$$
$$\{\text{Alice, Bob, Carol, Dave}\}\big\},$$
$$\mathcal{P}^2(V_0) = \mathcal{P}\big(\mathcal{P}^1(V_0)\big).$$

Select two 2-supervertices representing meta-teams:

$$v_1 = \big\{\{\text{Alice, Bob}\},\, \{\text{Bob, Carol}\}\big\},$$
$$v_2 = \big\{\{\text{Bob, Carol}\},\, \{\text{Carol, Dave}\}\big\}.$$

Thus

$$V = \{v_1,\, v_2\}, \qquad E = \big\{\{v_1,\, v_2\}\big\}.$$

Define the neutrosophic membership degrees for each meta-team $v \in V$:

$$T_V(v_1) = 0.85, \quad I_V(v_1) = 0.10, \quad F_V(v_1) = 0.05,$$
$$T_V(v_2) = 0.75, \quad I_V(v_2) = 0.20, \quad F_V(v_2) = 0.05.$$

For the single 2-superedge $e = \{v_1, v_2\}$, assign

$$T_E(e, v_1) = 0.80,\ I_E(e, v_1) = 0.15,\ F_E(e, v_1) = 0.05,$$
$$T_E(e, v_2) = 0.70,\ I_E(e, v_2) = 0.25,\ F_E(e, v_2) = 0.05.$$

One verifies:

$$0 \ \le\ T_V(v) + I_V(v) + F_V(v) \ \le\ 3,$$
$$0 \ \le\ T_E(e, v) + I_E(e, v) + F_E(e, v) \ \le\ 3,$$

and the appurtenance constraints

$$T_E(e, v) \le T_V(v),$$
$$I_E(e, v) \le I_V(v),$$
$$F_E(e, v) \le F_V(v),$$
$$\forall v \in e.$$

Therefore, the tuple

$$\big(V,\, E,\, T_V,\, I_V,\, F_V,\, T_E,\, I_E,\, F_E\big)$$

is a valid Single-valued Neutrosophic 2-Superhypergraph capturing both the hierarchical grouping of project teams and the uncertainty in their collaborative relationships.

D. Directed SuperHyperGraph

Directed HyperGraphs and Directed SuperHyperGraphs are graph classes that extend HyperGraphs and SuperHyperGraphs, respectively, in a manner analogous to Directed Graphs(cf.[92], [93]) . Below, we present their formal definitions and illustrative examples.

Definition 14 (Directed Hypergraph)**.** (cf.[92], [93]) A *directed hypergraph* is a pair

$$H = (V,\, E),$$

where

- V is a finite set of *vertices*.
- E is a finite set of *hyperarcs*, each hyperarc $e \in E$ being an ordered pair

  $$e = \big(T(e),\, H(e)\big) \in \mathcal{P}(V) \times \mathcal{P}(V),$$

  with

  $$T(e) \subseteq V,\ T(e) \neq \emptyset, \quad H(e) \subseteq V,\ H(e) \neq \emptyset.$$

Intuitively, each $e = (T(e), H(e))$ carries "flow" from all vertices in $T(e)$ (the *tail*) to all vertices in $H(e)$ (the *head*).

Example 15 (Directed Hypergraph on $V = \{v_1, v_2, v_3, v_4\}$)**.** Let the vertex set be

$$V = \{\, v_1,\, v_2,\, v_3,\, v_4\}.$$

Define two directed hyperarcs as follows:

- $e_1 = \big(T(e_1),\, H(e_1)\big)$, where

  $$T(e_1) = \{\, v_1,\, v_2\}, \qquad H(e_1) = \{\, v_3\}.$$

  Intuitively, e_1 carries "flow" from vertices v_1 and v_2 into v_3.
- $e_2 = \big(T(e_2),\, H(e_2)\big)$, where

  $$T(e_2) = \{\, v_2\}, \qquad H(e_2) = \{\, v_3,\, v_4\}.$$

  Here e_2 directs flow from v_2 to both v_3 and v_4.

Thus the hyperarc set is

$$E = \{\, e_1,\, e_2\},$$

and the pair

$$H = (V,\, E)$$

is a valid directed hypergraph.

Definition 16 (Directed n-SuperHyperGraph)**.** (cf.[82], [4], [19]) Let S be a nonempty *base set* and let $n \ge 0$ be an integer. Define iterated powersets by

$$\mathcal{P}^0(S) = S, \quad \mathcal{P}^{k+1}(S) = \mathcal{P}\big(\mathcal{P}^k(S)\big) \quad (k \ge 0).$$

A *directed n-SuperHyperGraph* is a pair

$$\mathrm{DSHG}^{(n)} = (V,\, E),$$

where

$$V \ \subseteq\ \mathcal{P}^n(S), \quad E \ \subseteq\ \mathcal{P}^n(S) \times \mathcal{P}^n(S),$$

and each directed n-superedge $e \in E$ is an ordered pair

$$e = \big(\mathrm{Tail}(e),\, \mathrm{Head}(e)\big), \quad \mathrm{Tail}(e),\, \mathrm{Head}(e) \subseteq \mathcal{P}^n(S),$$

typically both nonempty. Such an e carries "flow" from the entire set $\mathrm{Tail}(e)$ of n-supervertices into $\mathrm{Head}(e)$.

Example 17 (Directed 2-SuperHyperGraph on $S = \{a, b\}$)**.** Let the base set be

$$S = \{a, b\}.$$

Then

$$\mathcal{P}^1(S) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}, \qquad \mathcal{P}^2(S) = \mathcal{P}\big(\mathcal{P}^1(S)\big).$$

From $\mathcal{P}^2(S)$ select three 2-supervertices:

$$v_1 = \{\{a\}, \{a, b\}\}, \quad v_2 = \{\{b\}\}, \quad v_3 = \{\emptyset, \{b\}\}.$$

Thus

$$V = \{\, v_1,\, v_2,\, v_3\}.$$

Next, define two directed 2-superedges by

$$e_1 = \big(\{v_1,\, v_2\},\ \{v_3\}\big), \qquad e_2 = \big(\{v_2\},\ \{v_1,\, v_3\}\big).$$

Hence

$$E = \{\, e_1,\, e_2\}.$$

The pair

$$\mathrm{DSHG}^{(2)} \ =\ (V,\, E)$$

is a valid directed 2-SuperHyperGraph on the base set S.

E. Fuzzy Directed HyperGraph

A Fuzzy Directed HyperGraph is a concept that extends the Fuzzy Directed Graph by incorporating the structure of a HyperGraph (cf.[63], [64], [65], [66]).

Definition 18 (Fuzzy Directed Hypergraph)**.** (cf.[63], [64], [65], [66]) Let V be a nonempty set of *vertices*. A *fuzzy directed hypergraph* is a quadruple

$$H = \big(V,\ E,\ \sigma,\ \mu\big),$$

where

- E is a finite set of *directed hyperarcs*. Each $e \in E$ is an ordered pair $\big(T(e),\ H(e)\big)$ with

$$T(e)\ \subseteq\ V, \quad T(e) \neq \emptyset, \qquad H(e)\ \subseteq\ V \setminus T(e)$$

  called its *tail* and *head*, respectively.
- $\sigma : V \to [0,1]$ assigns to each vertex $v \in V$ a membership degree $\sigma(v)$.
- $\mu : E \to [0,1]$ assigns to each hyperarc $e \in E$ a membership degree $\mu(e)$.

These functions satisfy the *consistency constraint*

$$\mu(e)\ \leq\ \min_{x \in T(e)\,\cup\, H(e)}\ \sigma(x), \quad \forall\, e \in E.$$

Example 19 (Fuzzy Directed Hypergraph)**.** Let the vertex set be

$$V = \{\, v_1,\ v_2,\ v_3,\ v_4\}.$$

Define two directed hyperarcs:

$$\begin{aligned} e_1 &= \big(T(e_1), H(e_1)\big) \\ &= \big(\{v_1, v_2\},\ \{v_3\}\big), \\ e_2 &= \big(T(e_2), H(e_2)\big) \\ &= \big(\{v_2\},\ \{v_3, v_4\}\big). \end{aligned}$$

Thus

$$E = \{e_1,\ e_2\}.$$

Assign vertex-membership degrees $\sigma : V \to [0,1]$ by

$$\sigma(v_1) = 0.8, \quad \sigma(v_2) = 0.6,$$

$$\sigma(v_3) = 0.7, \quad \sigma(v_4) = 0.5.$$

Assign hyperarc-membership degrees $\mu : E \to [0,1]$ by

$$\mu(e_1) = 0.6, \quad \mu(e_2) = 0.5.$$

We check the consistency constraint $\mu(e) \leq \min_{x \in T(e) \cup H(e)}\, \sigma(x)$ for each $e \in E$:

$$\begin{aligned} \mu(e_1) = 0.6\ &\leq\ \min\{\sigma(v_1), \sigma(v_2), \sigma(v_3)\} \\ &= \min\{0.8, 0.6, 0.7\} = 0.6, \\ \mu(e_2) = 0.5\ &\leq\ \min\{\sigma(v_2), \sigma(v_3), \sigma(v_4)\} \\ &= \min\{0.6, 0.7, 0.5\} = 0.5. \end{aligned}$$

Therefore

$$\big(V,\ E,\ \sigma,\ \mu\big)$$

is a valid fuzzy directed hypergraph.

F. Single-valued Neutrosophic Directed HyperGraph

A Single-valued Neutrosophic Directed HyperGraph is a concept that extends the Single-valued Neutrosophic Directed Graph by incorporating the structure of a HyperGraph (cf.[67], [68], [69]).

Definition 20 (Single-valued Neutrosophic Directed Hypergraph)**.** (cf.[69]) A *single-valued neutrosophic directed hypergraph* on a nonempty set X is an ordered pair

$$G'\ =\ \big(G,\ \{F_j\}_{j=1}^{n}\big),$$

where

$$G\ =\ \{\, G_j\}_{j=1}^{n}, \quad G_j\ =\ \big(T(G_j),\, H(G_j)\big)$$

is a family of nontrivial single-valued neutrosophic subsets of X, with

$$T(G_j)\ =\ \{\, \big(v, \alpha_G(v), \beta_G(v), \gamma_G(v)\big)\ |\ v \in X\},$$

$$H(G_j)\ =\ \{\, \big(v', \alpha_G(v'), \beta_G(v'), \gamma_G(v')\big)\ |\ v' \in X\},$$

and each neutrosophic *hyperarc* is

$$F_j\big(T(G_j), H(G_j)\big) = (\alpha_{F_j},\, \beta_{F_j},\, \gamma_{F_j})$$

satisfying, for all j,

$$\begin{aligned} \alpha_{F_j}\ &\leq \bigwedge_{v \in T(G_j),\, v' \in H(G_j)} \big(\alpha_G(v) \wedge \alpha_G(v')\big), \\ \beta_{F_j}\ &\leq \bigwedge_{v \in T(G_j),\, v' \in H(G_j)} \big(\beta_G(v) \wedge \beta_G(v')\big), \\ \gamma_{F_j}\ &\leq \bigvee_{v \in T(G_j),\, v' \in H(G_j)} \big(\gamma_G(v) \wedge \gamma_G(v')\big), \end{aligned}$$

and

$$X\ =\ \bigcup_{j=1}^{n} \mathrm{supp}(G_j).$$

Example 21 (Single-valued Neutrosophic Directed Hypergraph on $X = \{v_1, v_2, v_3\}$)**.** Let the universe be

$$X = \{v_1,\ v_2,\ v_3\}.$$

Assign single-valued neutrosophic vertex-memberships by

$$\begin{aligned} &\alpha_G(v_1) = 0.7, \quad \beta_G(v_1) = 0.2, \quad \gamma_G(v_1) = 0.1, \\ &\alpha_G(v_2) = 0.6, \quad \beta_G(v_2) = 0.3, \quad \gamma_G(v_2) = 0.1, \\ &\alpha_G(v_3) = 0.8, \quad \beta_G(v_3) = 0.1, \quad \gamma_G(v_3) = 0.1, \end{aligned}$$

so that for each $v \in X$, $0 \leq \alpha_G(v) + \beta_G(v) + \gamma_G(v) \leq 3$.

Define two directed neutrosophic hyperarcs G_1, G_2:

$$G_1 : \begin{cases} T(G_1) = \{v_1, v_2\}, \\ H(G_1) = \{v_3\}, \end{cases}$$

$$G_2 : \begin{cases} T(G_2) = \{v_2\}, \\ H(G_2) = \{v_1, v_3\}. \end{cases}$$

Compute the admissible neutrosophic hyperarc-memberships:

For G_1:

$$\begin{aligned}\alpha_{F_1} &\le \min\Big\{\min\big(\alpha_G(v_1),\alpha_G(v_3)\big),\\ &\qquad \min\big(\alpha_G(v_2),\alpha_G(v_3)\big)\Big\}\\ &= \min\{0.7,0.6\} = 0.6,\\ \beta_{F_1} &\le \min\Big\{\min\big(\beta_G(v_1),\beta_G(v_3)\big),\\ &\qquad \min\big(\beta_G(v_2),\beta_G(v_3)\big)\Big\}\\ &= \min\{0.1,0.1\} = 0.1,\\ \gamma_{F_1} &\le \max\Big\{\min\big(\gamma_G(v_1),\gamma_G(v_3)\big),\\ &\qquad \min\big(\gamma_G(v_2),\gamma_G(v_3)\big)\Big\}\\ &= \max\{0.1,0.1\} = 0.1.\end{aligned}$$

Choose $\alpha_{F_1} = 0.6,\ \beta_{F_1} = 0.1,\ \gamma_{F_1} = 0.1.$

For G_2:

$$\begin{aligned}\alpha_{F_2} &\le \min\Big\{\min\big(\alpha_G(v_2),\alpha_G(v_1)\big),\\ &\qquad \min\big(\alpha_G(v_2),\alpha_G(v_3)\big)\Big\}\\ &= \min\{0.6,0.6\} = 0.6,\\ \beta_{F_2} &\le \min\Big\{\min\big(\beta_G(v_2),\beta_G(v_1)\big),\\ &\qquad \min\big(\beta_G(v_2),\beta_G(v_3)\big)\Big\}\\ &= \min\{0.2,0.1\} = 0.1,\\ \gamma_{F_2} &\le \max\Big\{\min\big(\gamma_G(v_2),\gamma_G(v_1)\big),\\ &\qquad \min\big(\gamma_G(v_2),\gamma_G(v_3)\big)\Big\}\\ &= \max\{0.1,0.1\} = 0.1.\end{aligned}$$

Choose $\alpha_{F_2} = 0.6,\ \beta_{F_2} = 0.1,\ \gamma_{F_2} = 0.1.$

Finally, note

$$\begin{aligned}X &= T(G_1)\cup H(G_1)\ \cup\ T(G_2)\cup H(G_2)\\ &= \{v_1,v_2,v_3\},\end{aligned}$$

so $\bigcup_{j=1}^{2}\operatorname{supp}(G_j) = X$. Therefore

$$\big(\{G_1,G_2\},\ \{F_1,F_2\}\big)$$

with the above membership assignments is a valid single-valued neutrosophic directed hypergraph on X.

III. Result: Fuzzy Directed n-Superhypergraph

We define the *Fuzzy Directed n-Superhypergraph* as an extension of the classical *Fuzzy Directed Hypergraph* by incorporating the hierarchical structure of n-Superhypergraphs.

Definition 22 (Fuzzy Directed n-Superhypergraph)**.** Let S be a nonempty *base set* and let $n \ge 0$ be an integer. Define iterated powersets by

$$\mathcal{P}^0(S) = S, \qquad \mathcal{P}^{k+1}(S) = \mathcal{P}\big(\mathcal{P}^k(S)\big) \quad (k \ge 0).$$

A *directed n-Superhypergraph* is a pair $\mathrm{DSHG}^{(n)} = (V,E)$ with

$$V \subseteq \mathcal{P}^n(S), \qquad E \subseteq \mathcal{P}^n(S)\times\mathcal{P}^n(S),$$

where each $e \in E$ is of the form $\big(\mathrm{Tail}(e),\mathrm{Head}(e)\big)$. A *fuzzy directed n-Superhypergraph* is then the quadruple

$$\big(V,\ E,\ \sigma,\ \mu\big),$$

where

- $\sigma : V \to [0,1]$ assigns to each n-supervertex v a membership degree $\sigma(v)$.
- $\mu : E \to [0,1]$ assigns to each directed n-superedge e a membership degree $\mu(e)$.

These satisfy the *edge-appurtenance constraint*

$$\mu(e) \ \le\ \min_{x\in\mathrm{Tail}(e)\,\cup\,\mathrm{Head}(e)}\sigma(x),$$
$$\forall\, e \in E.$$

We present several concrete examples from real-life scenarios. By employing such modeling approaches, effective decision-making can be achieved.

Example 23 (Fuzzy Directed 2-Superhypergraph for E-Commerce User Flow)**.** Let

$$S = \{\text{Browse, AddToCart}\},$$
$$\mathcal{P}^1(S) = \{\varnothing, \{\text{Browse}\}, \{\text{AddToCart}\},$$
$$\{\text{Browse, AddToCart}\}\},$$
$$\mathcal{P}^2(S) = \mathcal{P}\big(\mathcal{P}^1(S)\big).$$

Define two 2-supervertices representing user behavior patterns:

$$v_1 = \big\{\{\text{Browse}\}, \{\text{Browse, AddToCart}\}\big\},$$
$$v_2 = \{\{\text{AddToCart}\}\}.$$

Thus

$$V = \{v_1,\ v_2\}.$$

Next, model the directed transitions between these patterns:

$$e_1 = \big(\{v_1,v_2\},\ \{v_2\}\big), \quad e_2 = \big(\{v_2\},\ \{v_1\}\big).$$

Hence

$$E = \{e_1,\ e_2\}.$$

Assign membership degrees $\sigma : V \to [0,1]$ based on observed user frequencies:

$$\sigma(v_1) = 0.80, \quad \sigma(v_2) = 0.60.$$

Define the edge-membership $\mu : E \to [0,1]$ by

$$\mu(e_1) = 0.55, \quad \mu(e_2) = 0.45.$$

We verify the edge-appurtenance constraint $\mu(e) \le \min_{x\in T(e)\cup H(e)}\sigma(x)$:

$$\mu(e_1) = 0.55 \ \le\ \min\{0.80,\ 0.60\} = 0.60,$$
$$\mu(e_2) = 0.45 \ \le\ \min\{0.60,\ 0.80\} = 0.60.$$

Therefore,

$$(V,\ E,\ \sigma,\ \mu)$$

constitutes a valid fuzzy directed 2-superhypergraph, capturing key stages of user engagement in an online retail platform.

Example 24 (Fuzzy Directed 3-Superhypergraph in Corporate Program Management)**.** Let the base set of *tasks* be

$$S = \{\text{Design, Develop, Test}\}.$$

Then the iterated powersets are

$$\mathcal{P}^1(S) = \{\emptyset, \{\text{Design}\}, \{\text{Develop}\}, \{\text{Test}\},$$
$$\{\text{Design}, \text{Develop}\}, \dots\},$$
$$\mathcal{P}^2(S) = \mathcal{P}\big(\mathcal{P}^1(S)\big),$$
$$\mathcal{P}^3(S) = \mathcal{P}\big(\mathcal{P}^2(S)\big).$$

We interpret $\mathcal{P}^1(S)$ as *subprojects* (sets of tasks), $\mathcal{P}^2(S)$ as *projects* (sets of subprojects), and $\mathcal{P}^3(S)$ as *programs* (sets of projects).

From $\mathcal{P}^2(S)$ define two projects:

$$P_a = \big\{\{\text{Design}\}, \{\text{Develop}, \text{Test}\}\big\},$$
$$P_b = \big\{\{\text{Develop}\}, \{\text{Test}\}\big\}.$$

Then form two programs (3-supervertices) in $\mathcal{P}^3(S)$:

$$\text{Prog}_1 = \{P_a,\, P_b\}, \quad \text{Prog}_2 = \{P_b\}.$$

Thus

$$V = \{\text{Prog}_1,\ \text{Prog}_2\}.$$

Next, define two directed 3-superedges:

$$e_1 = \big(\{\text{Prog}_1\},\, \{\text{Prog}_2\}\big),$$
$$e_2 = \big(\{\text{Prog}_2\},\, \{\text{Prog}_1\}\big).$$

Hence

$$E = \{e_1,\, e_2\}.$$

Assign a fuzzy membership to each program:

$$\sigma(\text{Prog}_1) = 0.9, \quad \sigma(\text{Prog}_2) = 0.7.$$

Assign a fuzzy strength to each directed superedge:

$$\mu(e_1) = 0.6, \quad \mu(e_2) = 0.5.$$

We verify the edge-appurtenance constraint

$$\mu(e) \le \min_{x \in \text{Tail}(e) \cup \text{Head}(e)} \sigma(x)$$

:

$$\mu(e_1) = 0.6 \ \le\ \min\{\sigma(\text{Prog}_1), \sigma(\text{Prog}_2)\}$$
$$= \min\{0.9, 0.7\} = 0.7,$$
$$\mu(e_2) = 0.5 \ \le\ \min\{\sigma(\text{Prog}_2), \sigma(\text{Prog}_1)\}$$
$$= \min\{0.7, 0.9\} = 0.7.$$

Therefore

$$\big(V,\, E,\, \sigma,\, \mu\big)$$

is a valid fuzzy directed 3-Superhypergraph modeling the flow of information between corporate programs.

Example 25 (Decision-Making with Fuzzy Directed 2-Superhypergraph)**.** A manufacturer must choose between two supplier portfolios based on cost, reliability, and performance. Let the base set of evaluation criteria be

$$S = \{cost,\, reliability,\, performance\}.$$

We form the first and second iterated powersets:

$$\mathcal{P}^1(S) = \big\{\,\emptyset, \{cost\}, \{reliability\}, \{performance\},$$
$$\{cost, reliability\}, \dots\},$$
$$\mathcal{P}^2(S) = \mathcal{P}\big(\mathcal{P}^1(S)\big).$$

From $\mathcal{P}^2(S)$ we define two *alternatives* (2–supervertices):

$$v_1 = \big\{\{cost,\, reliability\}, \{performance\}\big\},$$
$$v_2 = \big\{\{cost\}, \{reliability,\, performance\}\big\}.$$

Thus

$$V = \{v_1,\, v_2\}.$$

We model the decision flow as a single directed 2–superedge:

$$e_1 = \big(\{v_1\},\, \{v_2\}\big), \quad E = \{e_1\},$$

indicating a preference or transition from portfolio v_1 to v_2.

Next, assign fuzzy viability degrees to each portfolio:

$$\sigma(v_1) = 0.85, \quad \sigma(v_2) = 0.75.$$

These reflect the manufacturer's confidence in each portfolio's overall suitability.

Then assign a fuzzy preference strength to the directed edge:

$$\mu(e_1) = 0.70.$$

This value represents the confidence in moving from v_1 to v_2.

We verify the edge-appurtenance constraint:

$$\mu(e_1) = 0.70 \ \le\ \min\{\sigma(v_1), \sigma(v_2)\}$$
$$= \min\{0.85, 0.75\} = 0.75.$$

Since the constraint holds, $\big(V,\, E,\, \sigma,\, \mu\big)$ forms a valid fuzzy directed 2–superhypergraph that captures the decision-making process in supplier portfolio selection.

Example 26 (Decision-Making in Product Line Management with a Fuzzy Directed 3-Superhypergraph)**.** A technology company must decide how to allocate its R&D budget among two competing product lines. The base set of features is

$$S = \{Design,\, Implementation,\, Testing\}.$$

First-level modules (subprojects) are

$$M_1 = \{Design,\, Implementation\},$$
$$M_2 = \{Implementation,\, Testing\},$$
$$M_3 = \{Design,\, Testing\}.$$

Second-level products are

$$P_a = \{M_1,\, M_2\}, \qquad P_b = \{M_2,\, M_3\}.$$

Third-level product lines (2 alternatives) are

$$\text{PL}_1 = \{P_a,\, P_b\}, \qquad \text{PL}_2 = \{P_b\}.$$

Thus the 3–supervertex set is

$$V = \{\text{PL}_1, \text{PL}_2\}.$$

The company considers two possible shifts of investment:

$$e_1 = \big(\{\text{PL}_1\},\, \{\text{PL}_2\}\big),$$
$$e_2 = \big(\{\text{PL}_2\},\, \{\text{PL}_1\}\big),$$

so

$$E = \{e_1,\, e_2\}.$$

Assign fuzzy viability degrees:

$$\sigma(\mathrm{PL}_1) = 0.80, \quad \sigma(\mathrm{PL}_2) = 0.65.$$

Assign fuzzy confidence in each shift:

$$\mu(e_1) = 0.60, \quad \mu(e_2) = 0.50.$$

We verify the edge-appurtenance constraint for e_1:

$$\mu(e_1) = 0.60 \ \leq \ \min\{0.80, 0.65\} = 0.65,$$

and similarly for e_2. Hence

$$\big(V,\, E,\, \sigma,\, \mu\big)$$

is a valid fuzzy directed 3-superhypergraph modeling the firm's decision to reallocate R&D resources between product lines under uncertainty.

Next, we present several mathematical theorems as follows.

Theorem 27 (Generalization of Directed n-Superhypergraph)**.** *Every directed n-superhypergraph $\mathrm{DSHG}^{(n)} = (V, E)$ can be viewed as a fuzzy directed n-superhypergraph by setting $\sigma(v) = 1$ for all $v \in V$ and $\mu(e) = 1$ for all $e \in E$.*

Proof: Let $\mathrm{DSHG}^{(n)} = (V, E)$ be given. Define $\sigma : V \to [0, 1]$ by $\sigma(v) = 1$ and $\mu : E \to [0, 1]$ by $\mu(e) = 1$. Then for each $e \in E$,

$$\min_{x \in \mathrm{Tail}(e) \cup \mathrm{Head}(e)} \sigma(x) = \min_{x \in \mathrm{Tail}(e) \cup \mathrm{Head}(e)} 1 = 1,$$

and so $\mu(e) = 1 \leq 1$. Hence $\big(V, E, \sigma, \mu\big)$ satisfies the constraint of Definition 22, yielding a fuzzy directed n-superhypergraph. ∎

Theorem 28 (Generalization of Fuzzy Directed Hypergraph)**.** *Every fuzzy directed hypergraph $\big(V, E, \sigma, \mu\big)$ is a fuzzy directed n-superhypergraph with $n = 0$.*

Proof: If $n = 0$, then $\mathcal{P}^0(S) = S$. A directed hypergraph (V, E) with $E \subseteq \mathcal{P}^0(S) \times \mathcal{P}^0(S)$ is exactly a directed 0-superhypergraph. Retaining the same $\sigma : V \to [0, 1]$ and $\mu : E \to [0, 1]$, the edge-appurtenance constraint $\mu(e) \leq \min_{x \in T(e) \cup H(e)} \sigma(x)$ coincides with that of Definition 22 for $n = 0$. Thus every fuzzy directed hypergraph is a special case of a fuzzy directed n-superhypergraph. ∎

Theorem 29 (Generalization of Fuzzy n-Superhypergraph)**.** *Every fuzzy n-superhypergraph $\big(V, E, \sigma, \mu\big)$ can be viewed as a fuzzy directed n-superhypergraph by replacing each undirected superedge $e \subseteq V$ with the directed superedge (e, e).*

Proof: Let $\big(V, E, \sigma, \mu\big)$ be a fuzzy n-superhypergraph, so $E \subseteq \mathcal{P}^n(S)$. Define

$$E' \ = \ \{\, \big(e, e\big) \mid e \in E\}$$
$$\subseteq \ \mathcal{P}^n(S) \times \mathcal{P}^n(S),$$

and keep $\sigma : V \to [0, 1]$ and define $\mu' : E' \to [0, 1]$ by $\mu'((e, e)) = \mu(e)$. For each $\tilde{e} = (e, e) \in E'$,

$$\min_{x \in \mathrm{Tail}(\tilde{e}) \cup \mathrm{Head}(\tilde{e})} \sigma(x) = \min_{x \in e} \sigma(x),$$

so $\mu'(\tilde{e}) = \mu(e) \leq \min_{x \in e} \sigma(x)$. Hence $\big(V, E', \sigma, \mu'\big)$ satisfies the fuzzy directed n-superhypergraph constraint, embedding the undirected case into the directed framework. ∎

Theorem 30 (α-Cut Substructure)**.** *Let $\mathcal{H} = (V, E, \sigma, \mu)$ be a fuzzy directed n-Superhypergraph as in Definition 22, and let $\alpha \in (0, 1]$. Define*

$$V_\alpha \ = \ \{\, v \in V \mid \sigma(v) \geq \alpha\},$$
$$E_\alpha \ = \ \{\, e \in E \mid \mu(e) \geq \alpha\}.$$

Then the pair (V_α, E_α) is a (crisp) directed n-Superhypergraph.

Proof: Let $e \in E_\alpha$. By definition $\mu(e) \geq \alpha$. Since $\mathcal{H}$ satisfies

$$\mu(e) \ \leq \ \min_{x \in \mathrm{Tail}(e) \cup \mathrm{Head}(e)} \sigma(x),$$

we have

$$\min_{x \in \mathrm{Tail}(e) \cup \mathrm{Head}(e)} \sigma(x) \ \geq \ \mu(e) \ \geq \ \alpha.$$

Hence for every $x \in \mathrm{Tail}(e) \cup \mathrm{Head}(e)$, $\sigma(x) \geq \alpha$, so $x \in V_\alpha$. Therefore $\mathrm{Tail}(e), \mathrm{Head}(e) \subseteq V_\alpha$, and (V_α, E_α) inherits the directed n-superhypergraph structure. ∎

Theorem 31 (Closure under Intersection)**.** *Let $\mathcal{H}_i = (V, E, \sigma_i, \mu_i)$ for $i = 1, 2$ be two fuzzy directed n-Superhypergraphs on the same V, E. Define*

$$\sigma_\wedge(v) = \min\{\sigma_1(v), \sigma_2(v)\},$$
$$\mu_\wedge(e) = \min\{\mu_1(e), \mu_2(e)\}.$$

Then $\mathcal{H}_\wedge = (V, E, \sigma_\wedge, \mu_\wedge)$ is also a fuzzy directed n-Superhypergraph.

Proof: For any $e \in E$, since each $\mathcal{H}_i$ satisfies $\mu_i(e) \leq \min_{x \in \mathrm{Tail}(e) \cup \mathrm{Head}(e)} \sigma_i(x)$, we have

$$\mu_\wedge(e) = \min\{\mu_1(e), \mu_2(e)\}$$
$$\leq \min\big\{\min_x \sigma_1(x), \ \min_x \sigma_2(x)\big\}$$
$$= \min_x \min\{\sigma_1(x), \sigma_2(x)\}$$
$$= \min_x \sigma_\wedge(x).$$

Thus $\mu_\wedge(e) \leq \min_{x \in \mathrm{Tail}(e) \cup \mathrm{Head}(e)} \sigma_\wedge(x)$ for all e, proving the edge-appurtenance constraint. ∎

Theorem 32 (Closure under Union)**.** *With the same notation as Theorem 50, define*

$$\sigma_\vee(v) = \max\{\sigma_1(v), \sigma_2(v)\},$$
$$\mu_\vee(e) = \max\{\mu_1(e), \mu_2(e)\}.$$

Then $\mathcal{H}_\vee = (V, E, \sigma_\vee, \mu_\vee)$ is a fuzzy directed n-Superhypergraph.

Proof: For any $e \in E$,

$$\mu_\vee(e) = \max\{\mu_1(e), \mu_2(e)\}$$
$$\leq \max\big\{\min_x \sigma_1(x), \ \min_x \sigma_2(x)\big\}$$

$$\leq \min_x \max\{\sigma_1(x), \sigma_2(x)\}$$
$$= \min_x \sigma_\vee(x),$$

where we use $\max\{\min a_i, \min b_i\} \leq \min\{\max a_i, \max b_i\}$. Hence the edge-appurtenance constraint holds for $\mathcal{H}_\vee$. ∎

Theorem 33 (Nestedness of α-cuts)**.** *Let $\mathcal{H} = (V, E, \sigma, \mu)$ be a fuzzy directed n-Superhypergraph and $0 \leq \beta \leq \alpha \leq 1$. Define*

$$V_\alpha = \{\, v \in V \mid \sigma(v) \geq \alpha\},$$
$$E_\alpha = \{\, e \in E \mid \mu(e) \geq \alpha\},$$

and similarly V_β, E_β. Then

$$V_\alpha \subseteq V_\beta, \quad E_\alpha \subseteq E_\beta.$$

Proof: If $\sigma(v) \geq \alpha$ and $\alpha \geq \beta$, then $\sigma(v) \geq \beta$, so $v \in V_\beta$. Likewise, $\mu(e) \geq \alpha$ implies $\mu(e) \geq \beta$, hence $e \in E_\beta$. ∎

Theorem 34 (Closure under Scalar Multiplication)**.** *Let $\mathcal{H} = (V, E, \sigma, \mu)$ be a fuzzy directed n-Superhypergraph and $\lambda \in [0, 1]$. Define*

$$\sigma_\lambda(v) = \lambda\, \sigma(v),$$
$$\mu_\lambda(e) = \lambda\, \mu(e).$$

Then $\mathcal{H}_\lambda = (V, E, \sigma_\lambda, \mu_\lambda)$ is also a fuzzy directed n-Superhypergraph.

Proof: For any $e \in E$,

$$\mu_\lambda(e) = \lambda\, \mu(e) \leq \lambda \min_{x \in \mathrm{Tail}(e) \cup \mathrm{Head}(e)} \sigma(x)$$
$$= \min_{x \in \mathrm{Tail}(e) \cup \mathrm{Head}(e)} (\lambda\, \sigma(x))$$
$$= \min_{x \in \mathrm{Tail}(e) \cup \mathrm{Head}(e)} \sigma_\lambda(x).$$

Thus the edge-appurtenance constraint holds. ∎

Theorem 35 (Convex Combination)**.** *Let $\mathcal{H}_1 = (V, E, \sigma_1, \mu_1)$ and $\mathcal{H}_2 = (V, E, \sigma_2, \mu_2)$ be two fuzzy directed n-Superhypergraphs and $\alpha \in [0, 1]$. Define*

$$\sigma'(v) = \alpha\, \sigma_1(v) + (1 - \alpha)\, \sigma_2(v),$$
$$\mu'(e) = \alpha\, \mu_1(e) + (1 - \alpha)\, \mu_2(e).$$

Then $\mathcal{H}' = (V, E, \sigma', \mu')$ is a fuzzy directed n-Superhypergraph.

Proof: For any $e \in E$,

$$\mu'(e) = \alpha\mu_1(e) + (1 - \alpha)\mu_2(e)$$
$$\leq \alpha \min_x \sigma_1(x) + (1 - \alpha) \min_x \sigma_2(x)$$
$$\leq \min_x \big(\alpha\sigma_1(x) + (1 - \alpha)\sigma_2(x)\big) = \min_x \sigma'(x).$$

Hence the appurtenance constraint is satisfied. ∎

Theorem 36 (Support Substructure)**.** *Let $\mathcal{H} = (V, E, \sigma, \mu)$ be a fuzzy directed n-Superhypergraph. Define*

$$\mathrm{supp}(V) = \{\, v \in V \mid \sigma(v) > 0\},$$
$$\mathrm{supp}(E) = \{\, e \in E \mid \mu(e) > 0\}.$$

Then

$$\big(\mathrm{supp}(V),\, \mathrm{supp}(E)\big)$$

is a (crisp) directed n-Superhypergraph.

Proof: If $e \in \mathrm{supp}(E)$ then $\mu(e) > 0$. By the appurtenance constraint,

$$0 < \mu(e) \leq \min_{x \in \mathrm{Tail}(e) \cup \mathrm{Head}(e)} \sigma(x),$$

so $\sigma(x) > 0$ for all $x \in \mathrm{Tail}(e) \cup \mathrm{Head}(e)$. Thus every such x lies in $\mathrm{supp}(V)$, and the directed n-superhypergraph structure is inherited. ∎

Theorem 37 (Underlying Undirected Hypergraph)**.** *Let $\mathcal{H} = (V, E, \sigma, \mu)$ be a fuzzy directed n-Superhypergraph. Define the undirected edge-set*

$$E_U = \{\mathrm{Tail}(e) \cup \mathrm{Head}(e) \mid e \in E\}$$

and a membership function $\mu_U : E_U \to [0, 1]$ by $\mu_U(e_U) = \mu(e)$ whenever $e_U = \mathrm{Tail}(e) \cup \mathrm{Head}(e)$. Then $\big(V, E_U, \sigma, \mu_U\big)$ is a fuzzy n-Superhypergraph.

Proof: For each $e_U \in E_U$ coming from $e \in E$,

$$\mu_U(e_U) = \mu(e) \leq \min_{x \in \mathrm{Tail}(e) \cup \mathrm{Head}(e)} \sigma(x)$$
$$= \min_{x \in e_U} \sigma(x),$$

so the fuzzy-appurtenance constraint holds in the undirected setting. ∎

IV. Result: Single-valued Neutrosophic Directed n-Superhypergraph

We define the *Single-valued Neutrosophic Directed n-Superhypergraph* as an extension of the classical *Single-valued Neutrosophic Directed Hypergraph* by incorporating the hierarchical structure of n-Superhypergraphs.

Definition 38 (Neutrosophic Directed Superhypergraph)**.** Let S be a nonempty *base set* and $n \geq 0$ an integer. Define

$$\mathcal{P}^0(S) = S,$$
$$\mathcal{P}^{k+1}(S) = \mathcal{P}(\mathcal{P}^k(S)) \quad (k \geq 0).$$

A *directed n-Superhypergraph* is a pair $\mathrm{DSHG}^{(n)} = (V, E)$ with

$$V \subseteq \mathcal{P}^n(S),$$
$$E \subseteq \mathcal{P}^n(S) \times \mathcal{P}^n(S),$$

where each $e \in E$ is $\big(\mathrm{Tail}(e), \mathrm{Head}(e)\big)$. A *single-valued neutrosophic directed n-Superhypergraph* is the septuple

$$\big(V,\, E,\, T_V,\, I_V,\, F_V,\, T_E,\, I_E,\, F_E\big),$$

where

$$T_V,\, I_V,\, F_V : V \to [0, 1],$$
$$T_V(v) + I_V(v) + F_V(v) \leq 3,\ \forall v \in V,$$
$$T_E,\, I_E,\, F_E : E \to [0, 1],$$
$$T_E(e) \leq \min_{x \in \mathrm{Tail}(e) \cup \mathrm{Head}(e)} T_V(x),$$
$$I_E(e) \leq \min_{x \in \mathrm{Tail}(e) \cup \mathrm{Head}(e)} I_V(x),$$
$$F_E(e) \leq \min_{x \in \mathrm{Tail}(e) \cup \mathrm{Head}(e)} F_V(x), \quad \forall e \in E.$$

We present several concrete examples from real-life scenarios. By employing such modeling approaches, effective decision-making can be achieved.

Example 39 (Single-valued Neutrosophic Directed 2-Superhypergraph for a Data-Driven Workflow)**.** Let the base set of *workflow phases* be

$$S = \{\text{Data Collection, Data Analysis}\}.$$

Then

$$\mathcal{P}^1(S) = \{\emptyset, \{\text{Data Collection}\}, \{\text{Data Analysis}\}, \{\text{Data Collection}, \text{Data Analysis}\}\},$$

$$\mathcal{P}^2(S) = \mathcal{P}\big(\mathcal{P}^1(S)\big),$$

whose elements are all subsets of the four first-level subsets. From $\mathcal{P}^2(S)$, select two 2-supervertices:

$$v_1 = \big\{\{\text{Data Collection}\}, \{\text{Data Collection, Data Analysis}\}\big\},$$

$$v_2 = \{\{\text{Data Analysis}\}\}.$$

Thus the 2-supervertex set is

$$V = \{\, v_1,\, v_2\}.$$

Next, define two directed 2-superedges:

$$e_1 = \big(\{v_1\},\, \{v_2\}\big),$$

$$e_2 = \big(\{v_2\},\, \{v_1\}\big).$$

Hence

$$E = \{\, e_1,\, e_2\}.$$

Assign to each 2-supervertex v its truth, indeterminacy, and falsity degrees:

$$T_V(v_1) = 0.9, \quad I_V(v_1) = 0.05, \quad F_V(v_1) = 0.05,$$

$$T_V(v_2) = 0.8, \quad I_V(v_2) = 0.10, \quad F_V(v_2) = 0.10.$$

For each directed superedge e, assign:

$$T_E(e_1) = 0.8, \quad I_E(e_1) = 0.05, \quad F_E(e_1) = 0.05,$$

$$T_E(e_2) = 0.7, \quad I_E(e_2) = 0.05, \quad F_E(e_2) = 0.05.$$

We verify the appurtenance constraints. For e_1:

$$T_E(e_1) = 0.8 \;\le\; \min\{T_V(v_1), T_V(v_2)\} = \min\{0.9, 0.8\} = 0.8,$$

$$I_E(e_1) = 0.05 \;\le\; \min\{I_V(v_1), I_V(v_2)\} = \min\{0.05, 0.10\} = 0.05,$$

$$F_E(e_1) = 0.05 \;\le\; \min\{F_V(v_1), F_V(v_2)\} = \min\{0.05, 0.10\} = 0.05.$$

Similarly for e_2:

$$T_E(e_2) = 0.7 \;\le\; \min\{0.8, 0.9\} = 0.8,$$

$$I_E(e_2) = 0.05 \;\le\; \min\{0.10, 0.05\} = 0.05,$$

$$F_E(e_2) = 0.05 \;\le\; \min\{0.10, 0.05\} = 0.05.$$

Therefore

$$\big(V,\, E,\, T_V,\, I_V,\, F_V,\, T_E,\, I_E,\, F_E\big)$$

is a valid single-valued neutrosophic directed 2-superhypergraph modeling a simple data-driven workflow with feedback.

Example 40 (Single-valued Neutrosophic Directed 3-Superhypergraph in Multi-Program Management)**.** Let the base set of *tasks* be

$$S = \{\text{Design, Develop, Test}\}.$$

Then

$$\mathcal{P}^1(S) = \{\emptyset, \{\text{Design}\}, \{\text{Develop}\}, \{\text{Test}\}, \dots\},$$

$$\mathcal{P}^2(S) = \mathcal{P}(\mathcal{P}^1(S)),$$

$$\mathcal{P}^3(S) = \mathcal{P}(\mathcal{P}^2(S)).$$

Interpret $\mathcal{P}^1(S)$ as *subprojects*, $\mathcal{P}^2(S)$ as *projects*, and $\mathcal{P}^3(S)$ as *programs*.

From $\mathcal{P}^2(S)$ define two projects:

$$P_a = \{\{\text{Design}\},\, \{\text{Develop}, \text{Test}\}\},$$

$$P_b = \{\{\text{Develop}\},\, \{\text{Test}\}\}.$$

Then form two programs (3-supervertices) in $\mathcal{P}^3(S)$:

$$\text{Prog}_1 = \{P_a,\, P_b\},$$

$$\text{Prog}_2 = \{P_b\}.$$

Thus

$$V = \{\text{Prog}_1,\, \text{Prog}_2\}.$$

Next, define two directed 3-superedges:

$$e_1 = \big(\{\text{Prog}_1\},\, \{\text{Prog}_2\}\big),$$

$$e_2 = \big(\{\text{Prog}_2\},\, \{\text{Prog}_1\}\big),$$

so

$$E = \{e_1,\, e_2\}.$$

Assign to each program its single-valued neutrosophic vertex-memberships:

$$\begin{aligned} T_V(\text{Prog}_1) &= 0.9, \\ I_V(\text{Prog}_1) &= 0.05, \\ F_V(\text{Prog}_1) &= 0.05, \\ T_V(\text{Prog}_2) &= 0.8, \\ I_V(\text{Prog}_2) &= 0.10, \\ F_V(\text{Prog}_2) &= 0.10. \end{aligned}$$

These satisfy $T_V + I_V + F_V \le 3$. For each directed hyperarc $e \in E$, define:

$$T_E(e_1) = 0.8, \quad I_E(e_1) = 0.05, \quad F_E(e_1) = 0.05,$$

$$T_E(e_2) = 0.7, \quad I_E(e_2) = 0.05, \quad F_E(e_2) = 0.05.$$

We check the appurtenance constraints. For e_1:

$$T_E(e_1) = 0.8 \le \min\{T_V(\text{Prog}_1), T_V(\text{Prog}_2)\} = \min\{0.9, 0.8\} = 0.8,$$

$$I_E(e_1) = 0.05 \le \min\{I_V(\text{Prog}_1), I_V(\text{Prog}_2)\} = \min\{0.05, 0.10\} = 0.05,$$

$$F_E(e_1) = 0.05 \le \min\{F_V(\text{Prog}_1), F_V(\text{Prog}_2)\} = \min\{0.05, 0.10\} = 0.05.$$

Similarly for e_2:

$$T_E(e_2) = 0.7 \leq \min\{0.8, 0.9\} = 0.8,$$
$$I_E(e_2) = 0.05 \leq \min\{0.10, 0.05\} = 0.05,$$
$$F_E(e_2) = 0.05 \leq \min\{0.10, 0.05\} = 0.05.$$

Finally, since

$$\operatorname{supp}(\mathrm{Prog}_1) \cup \operatorname{supp}(\mathrm{Prog}_2) = \{\mathrm{Prog}_1, \mathrm{Prog}_2\}$$

, the condition $\bigcup_{e \in E} \operatorname{supp}(e) = V$ holds. Therefore

$$\big(V,\, E,\, T_V,\, I_V,\, F_V,\, T_E,\, I_E,\, F_E\big)$$

is a valid single-valued neutrosophic directed 3-Superhypergraph modeling inter-program influence in a corporate portfolio.

Example 41 (Clinical Treatment Selection via Single-valued Neutrosophic Directed 2-Superhypergraph)**.** A hospital must choose between two composite treatment strategies based on cost, efficacy, and safety. Let the base set of criteria be

$$S = \{cost,\, efficacy,\, safety\}.$$

First-level criterion-sets (hypervertices) include, for example, $\{cost,\, efficacy\}$, $\{efficacy,\, safety\}$, $\{cost,\, safety\}$, etc. From these we form two second-level alternatives (2–supervertices):

$$v_1 = \big\{\{cost,\, efficacy\},\, \{safety\}\big\},$$
$$v_2 = \big\{\{cost\},\, \{efficacy,\, safety\}\big\}.$$

Thus

$$V = \{v_1,\, v_2\}.$$

We represent a directional recommendation from v_1 to v_2 as the single directed 2–superedge

$$e = \big(\{v_1\},\, \{v_2\}\big), \quad E = \{e\}.$$

Next, we assess each alternative under neutrosophic uncertainty:

$$T_V(v_1) = 0.8, \quad I_V(v_1) = 0.1, \quad F_V(v_1) = 0.1,$$
$$T_V(v_2) = 0.7, \quad I_V(v_2) = 0.2, \quad F_V(v_2) = 0.1.$$

Here T_V is confidence in the alternative, I_V reflects conflicting evidence, and F_V is confidence against.

For the recommendation edge, we assign:

$$T_E(e) = 0.65, \quad I_E(e) = 0.10, \quad F_E(e) = 0.05.$$

We verify the neutrosophic appurtenance constraints:

$$T_E(e) = 0.65 \leq \min\{T_V(v_1), T_V(v_2)\} = \min\{0.8, 0.7\} = 0.7,$$
$$I_E(e) = 0.10 \leq \min\{I_V(v_1), I_V(v_2)\} = \min\{0.1, 0.2\} = 0.1,$$
$$F_E(e) = 0.05 \leq \min\{F_V(v_1), F_V(v_2)\} = \min\{0.1, 0.1\} = 0.1.$$

Thus

$$\big(V,\, E,\, T_V,\, I_V,\, F_V,\, T_E,\, I_E,\, F_E\big)$$

is a valid single-valued neutrosophic directed 2–superhypergraph that captures the hospital's decision-making process under uncertainty.

Example 42 (IT Strategy Selection via Single-valued Neutrosophic Directed 3-Superhypergraph)**.** A company must decide between two high-level IT initiatives, each composed of multiple projects in cloud adoption, AI integration, and cybersecurity. Let the base set be

$$S = \{Cloud,\, AI,\, Cybersecurity\}.$$

First-level modules (subprojects) include

$$M_1 = \{Cloud,\, AI\},$$
$$M_2 = \{AI,\, Cybersecurity\},$$
$$M_3 = \{Cloud,\, Cybersecurity\}.$$

Second-level products are

$$P_A = \{M_1,\, M_2\}, \qquad P_B = \{M_2,\, M_3\}.$$

Third-level initiatives (3–supervertices) are

$$\text{Alpha} = \{P_A,\, P_B\}, \qquad \text{Beta} = \{P_B\}.$$

Thus

$$V = \{\text{Alpha, Beta}\}.$$

We model the directional preference between initiatives by two directed superedges:

$$e_1 = (\{\text{Alpha}\},\, \{\text{Beta}\}),$$
$$e_2 = (\{\text{Beta}\},\, \{\text{Alpha}\}),$$

so

$$E = \{e_1,\, e_2\}.$$

Next, assign neutrosophic membership degrees to each initiative:

$$T_V(\text{Alpha}) = 0.85,$$
$$I_V(\text{Alpha}) = 0.10,$$
$$F_V(\text{Alpha}) = 0.05,$$
$$T_V(\text{Beta}) = 0.75,$$
$$I_V(\text{Beta}) = 0.15,$$
$$F_V(\text{Beta}) = 0.10.$$

Here T_V is the confidence in success, I_V the indeterminacy, and F_V the confidence against.

For the directed recommendations, we set:

$$T_E(e_1) = 0.70, \quad I_E(e_1) = 0.10, \quad F_E(e_1) = 0.05,$$
$$T_E(e_2) = 0.65, \quad I_E(e_2) = 0.10, \quad F_E(e_2) = 0.05.$$

We verify the neutrosophic appurtenance constraints. For e_1:

$$T_E(e_1) = 0.70 \;\leq\; \min\{T_V(\text{Alpha}), T_V(\text{Beta})\} = \min\{0.85, 0.75\} = 0.75,$$
$$I_E(e_1) = 0.10 \;\leq\; \min\{I_V(\text{Alpha}), I_V(\text{Beta})\} = \min\{0.10, 0.15\} = 0.10,$$
$$F_E(e_1) = 0.05 \;\leq\; \min\{F_V(\text{Alpha}), F_V(\text{Beta})\} = \min\{0.05, 0.10\} = 0.05.$$

An analogous check holds for e_2.

Therefore

$$\big(V,\, E,\, T_V,\, I_V,\, F_V,\, T_E,\, I_E,\, F_E\big)$$

is a valid single-valued neutrosophic directed 3-superhypergraph representing the company's strategic decision-making under uncertainty.

Example 43 (Single-valued Neutrosophic Directed 4-Superhypergraph for Enterprise IT Portfolio Strategy)**.** Let the base set of *technology domains* be

$$S = \{\mathit{Cloud},\ \mathit{AI},\ \mathit{Cybersecurity}\}.$$

Then

$$\mathcal{P}^1(S) = \{\emptyset, \{\mathit{Cloud}\}, \{\mathit{AI}\}, \{\mathit{Cybersecurity}\}, \ldots,$$
$$\{\mathit{Cloud}, \mathit{AI}, \mathit{Cybersecurity}\}\},$$
$$\mathcal{P}^2(S) = \mathcal{P}\big(\mathcal{P}^1(S)\big),$$
$$\mathcal{P}^3(S) = \mathcal{P}\big(\mathcal{P}^2(S)\big),$$
$$\mathcal{P}^4(S) = \mathcal{P}\big(\mathcal{P}^3(S)\big).$$

Define three first-level modules:

$$M_1 = \{\mathit{Cloud}, \mathit{AI}\},$$
$$M_2 = \{\mathit{AI}, \mathit{Cybersecurity}\},$$
$$M_3 = \{\mathit{Cloud}, \mathit{Cybersecurity}\}.$$

From these, form two second-level solutions:

$$P_A = \{M_1,\, M_2\}, \quad P_B = \{M_2,\, M_3\}.$$

Next, build two third-level initiatives:

$$I_1 = \{P_A,\, P_B\}, \quad I_2 = \{P_B\}.$$

Finally, select two fourth-level portfolios:

$$\Pi_1 = \{I_1,\, I_2\}, \quad \Pi_2 = \{I_2\}.$$

Thus

$$V = \{\Pi_1,\, \Pi_2\},$$
$$E = \{\, e_1 = (\{\Pi_1\}, \{\Pi_2\}),\ e_2 = (\{\Pi_2\}, \{\Pi_1\})\}.$$

Assign neutrosophic vertex-memberships:

$$T_V(\Pi_1) = 0.85,\ I_V(\Pi_1) = 0.10,\ F_V(\Pi_1) = 0.05,$$
$$T_V(\Pi_2) = 0.75,\ I_V(\Pi_2) = 0.15,\ F_V(\Pi_2) = 0.10.$$

Assign neutrosophic edge-memberships:

$$T_E(e_1) = 0.75,\ I_E(e_1) = 0.10,\ F_E(e_1) = 0.05,$$
$$T_E(e_2) = 0.70,\ I_E(e_2) = 0.10,\ F_E(e_2) = 0.10.$$

We verify for each $e_i \in E$:

$$T_E(e_i) \le \min_{x \in \mathrm{Tail}(e_i) \cup \mathrm{Head}(e_i)} T_V(x),$$
$$I_E(e_i) \le \min_{x \in \mathrm{Tail}(e_i) \cup \mathrm{Head}(e_i)} I_V(x),$$
$$F_E(e_i) \le \min_{x \in \mathrm{Tail}(e_i) \cup \mathrm{Head}(e_i)} F_V(x).$$

Hence,

$$\big(V,\, E,\, T_V,\, I_V,\, F_V,\, T_E,\, I_E,\, F_E\big)$$

is a valid single-valued neutrosophic directed 4-Superhypergraph modeling an enterprise IT portfolio strategy under uncertainty.

Example 44 (Single-valued Neutrosophic Directed 4-Superhypergraph for Corporate R&D Portfolio Decision)**.** Let the base set of *technology domains* be

$$S = \{\mathit{AI},\ \mathit{IoT},\ \mathit{Blockchain}\}.$$

Then

$$\mathcal{P}^1(S) = \{\varnothing, \{\mathit{AI}\}, \{\mathit{IoT}\},$$
$$\{\mathit{Blockchain}\}, \ldots, \{\mathit{AI}, \mathit{IoT}, \mathit{Blockchain}\}\},$$
$$\mathcal{P}^2(S) = \mathcal{P}(\mathcal{P}^1(S)),$$
$$\mathcal{P}^3(S) = \mathcal{P}(\mathcal{P}^2(S)),$$
$$\mathcal{P}^4(S) = \mathcal{P}(\mathcal{P}^3(S)).$$

Build the following hierarchy:

- *Modules* (1–supervertices):

$$M_1 = \{\mathit{AI}, \mathit{IoT}\},$$
$$M_2 = \{\mathit{IoT}, \mathit{Blockchain}\},$$
$$M_3 = \{\mathit{AI}, \mathit{Blockchain}\}.$$

- *Projects* (2–supervertices):

$$P_A = \{M_1,\, M_2\}, \quad P_B = \{M_2,\, M_3\}.$$

- *Initiatives* (3–supervertices):

$$I_1 = \{P_A,\, P_B\}, \quad I_2 = \{P_B\}.$$

- *Portfolios* (4–supervertices):

$$\Pi_1 = \{I_1,\, I_2\}, \quad \Pi_2 = \{I_2\}.$$

Thus

$$V = \{\Pi_1,\, \Pi_2\}, \quad E = \{\, e = (\{\Pi_1\}, \{\Pi_2\})\}.$$

Assign single-valued neutrosophic vertex degrees:

$$T_V(\Pi_1) = 0.90,\ I_V(\Pi_1) = 0.07,\ F_V(\Pi_1) = 0.03,$$
$$T_V(\Pi_2) = 0.75,\ I_V(\Pi_2) = 0.15,\ F_V(\Pi_2) = 0.10.$$

For the directed superedge e, set:

$$T_E(e) = 0.85, \quad I_E(e) = 0.10, \quad F_E(e) = 0.05.$$

We verify the neutrosophic appurtenance constraints:

$$T_E(e) = 0.85 \le \min\{T_V(\Pi_1), T_V(\Pi_2)\}$$
$$= \min\{0.90, 0.75\} = 0.75,$$
$$I_E(e) = 0.10 \le \min\{I_V(\Pi_1), I_V(\Pi_2)\}$$
$$= \min\{0.07, 0.15\} = 0.07,$$
$$F_E(e) = 0.05 \le \min\{F_V(\Pi_1), F_V(\Pi_2)\}$$
$$= \min\{0.03, 0.10\} = 0.03.$$

Therefore,

$$\big(V,\, E,\, T_V,\, I_V,\, F_V,\, T_E,\, I_E,\, F_E\big)$$

is a valid single-valued neutrosophic directed 4-superhypergraph capturing the decision-making process for selecting an R&D portfolio under uncertainty.

Next, we present several mathematical theorems as follows.

Theorem 45. *(*Generalizes Fuzzy Directed n-Superhypergraph*) If in Definition 38 we set*

$$I_V(v) = F_V(v) = 0, \quad I_E(e) = F_E(e) = 0$$
$$\forall v \in V, e \in E,$$

and let $\sigma(v) = T_V(v)$, $\mu(e) = T_E(e)$, then $\big(V, E, \sigma, \mu\big)$ is exactly a fuzzy directed n-superhypergraph.

Proof: With $I_V = F_V = I_E = F_E = 0$, the neutrosophic structure collapses to two functions $\sigma = T_V$ and $\mu = T_E$. The constraint $\mu(e) \leq \min_{x \in \mathrm{Tail}(e) \cup \mathrm{Head}(e)} \sigma(x)$ matches the fuzzy-directed-n-superhypergraph edge-appurtenance requirement. ■

Theorem 46. *(*Generalizes Single-valued Neutrosophic Directed Hypergraph*) When $n = 0$, $\mathcal{P}^0(S) = S$ and $\mathrm{Tail}(e), \mathrm{Head}(e) \subseteq S$. Then any single-valued neutrosophic directed 0-superhypergraph is exactly a single-valued neutrosophic directed hypergraph on S.*

Proof: Setting $n = 0$ reduces the vertex-set to $V \subseteq S$ and edges to ordered pairs in $S \times S$. All functions (T_V, I_V, F_V) on V and (T_E, I_E, F_E) on E remain, with identical appurtenance constraints. This is precisely the classical single-valued neutrosophic directed hypergraph. ■

Theorem 47. *(*Generalizes Single-valued Neutrosophic n-Superhypergraph*) Let $\big(V, E_U, T_V, I_V, F_V, T_E^U, I_E^U, F_E^U\big)$ be a single-valued neutrosophic n-superhypergraph with $E_U \subseteq \mathcal{P}^n(S)$. Define*

$$E_D = \{(e, e) \mid e \in E_U\} \subseteq \mathcal{P}^n(S) \times \mathcal{P}^n(S),$$

and extend

$$T_E(e, e) = T_E^U(e), \; I_E(e, e) = I_E^U(e),$$
$$F_E(e, e) = F_E^U(e).$$

Then $\big(V, E_D, T_V, I_V, F_V, T_E, I_E, F_E\big)$ is a single-valued neutrosophic directed n-superhypergraph.

Proof: Each undirected superedge e becomes directed pair (e, e). The vertex-memberships (T_V, I_V, F_V) remain unchanged. For each (e, e), the appurtenance constraints

$$T_E(e, e) = T_E^U(e) \leq \min_{x \in e} T_V(x)$$
$$= \min_{x \in \mathrm{Tail}(e,e) \cup \mathrm{Head}(e,e)} T_V(x),$$

(and similarly for I and F) hold by hypothesis on the undirected case. Thus all neutrosophic constraints are satisfied. ■

Theorem 48 (Truth α-cut Substructure)**.** *Let $\mathcal{N} = (V, E, T_V, I_V, F_V, T_E, I_E, F_E)$ be a single-valued neutrosophic directed n-Superhypergraph as in Definition 38, and let $\alpha \in (0, 1]$. Define*

$$V_T^\alpha = \{\, v \in V \mid T_V(v) \geq \alpha\},$$
$$E_T^\alpha = \{\, e \in E \mid T_E(e) \geq \alpha\}.$$

Then $\big(V_T^\alpha,\, E_T^\alpha\big)$ is a (crisp) directed n-Superhypergraph.

Proof: Take any $e \in E_T^\alpha$. By definition $T_E(e) \geq \alpha$. From the appurtenance constraint

$$T_E(e) \;\leq\; \min_{x \in \mathrm{Tail}(e) \cup \mathrm{Head}(e)} T_V(x),$$

we deduce

$$\min_{x \in \mathrm{Tail}(e) \cup \mathrm{Head}(e)} T_V(x)$$
$$\geq\; T_E(e) \;\geq\; \alpha.$$

Thus every

$$x \in \mathrm{Tail}(e) \cup \mathrm{Head}(e)$$

satisfies

$$T_V(x) \geq \alpha$$

, so lies in V_T^α. Hence

$$\mathrm{Tail}(e), \mathrm{Head}(e) \subseteq V_T^\alpha$$

, and

$$(V_T^\alpha, E_T^\alpha)$$

inherits the structure of a directed n-Superhypergraph. ■

Theorem 49 (Support Substructure)**.** *With $\mathcal{N}$ as above, define the supports*

$$\mathrm{supp}_V = \{\, v \in V \mid T_V(v) + I_V(v) + F_V(v) > 0\},$$
$$\mathrm{supp}_E = \{\, e \in E \mid T_E(e) + I_E(e) + F_E(e) > 0\}.$$

Then $\big(\mathrm{supp}_V,\quad \mathrm{supp}_E\big)$ is a (crisp) directed n-Superhypergraph.

Proof: Let $e \in \mathrm{supp}_E$, so $T_E(e) + I_E(e) + F_E(e) > 0$. Then at least one of $T_E(e), I_E(e), F_E(e)$ is positive. Suppose, for instance, $T_E(e) > 0$. From

$$T_E(e) \;\leq\; \min_{x \in \mathrm{Tail}(e) \cup \mathrm{Head}(e)} T_V(x),$$

we get $T_V(x) \geq T_E(e) > 0$ for all $x \in \mathrm{Tail}(e) \cup \mathrm{Head}(e)$, so $T_V(x) + I_V(x) + F_V(x) > 0$. The same argument applies if $I_E(e) > 0$ or $F_E(e) > 0$. Thus every $x \in \mathrm{Tail}(e) \cup \mathrm{Head}(e)$ lies in supp_V, giving the desired substructure. ■

Theorem 50 (Intersection of Two Neutrosophic Structures)**.** *Let $\mathcal{N}_i = (V, E, T_V^i, I_V^i, F_V^i, T_E^i, I_E^i, F_E^i)$ for $i = 1, 2$ be two single-valued neutrosophic directed n-Superhypergraphs on the same (V, E). Define*

$$T_V(v) = \min\{T_V^1(v), T_V^2(v)\},$$
$$I_V(v) = \max\{I_V^1(v), I_V^2(v)\},$$
$$F_V(v) = \max\{F_V^1(v), F_V^2(v)\},$$
$$T_E(e) = \min\{T_E^1(e), T_E^2(e)\},$$
$$I_E(e) = \max\{I_E^1(e), I_E^2(e)\},$$
$$F_E(e) = \max\{F_E^1(e), F_E^2(e)\}.$$

Then $\mathcal{N} = (V, E, T_V, I_V, F_V, T_E, I_E, F_E)$ is also a single-valued neutrosophic directed n-Superhypergraph.

Proof: First, for each $v \in V$,

$$T_V(v) + I_V(v) + F_V(v)$$

$$= \min(T_V^1, T_V^2) + \max(I_V^1, I_V^2) + \max(F_V^1, F_V^2)$$
$$\leq T_V^i(v) + I_V^i(v) + F_V^i(v) \leq 3 \quad (i = 1, 2),$$

so the vertex-sum constraint holds. Similarly for edges one checks

$$T_E(e) \leq \min\{\min_x T_V^1(x), \min_x T_V^2(x)\}$$
$$= \min_x \min\{T_V^1(x), T_V^2(x)\} = \min_x T_V(x),$$

and analogously for I_E, F_E. Hence all appurtenance constraints are satisfied. ∎

Theorem 51 (Union of Two Neutrosophic Structures)**.** *With the same notation as Theorem 50, define*

$$T_V(v) = \max\{T_V^1(v), T_V^2(v)\},$$
$$I_V(v) = \min\{I_V^1(v), I_V^2(v)\},$$
$$F_V(v) = \min\{F_V^1(v), F_V^2(v)\},$$
$$T_E(e) = \max\{T_E^1(e), T_E^2(e)\},$$
$$I_E(e) = \min\{I_E^1(e), I_E^2(e)\},$$
$$F_E(e) = \min\{F_E^1(e), F_E^2(e)\}.$$

Then

$$\mathcal{N} = (V, E, T_V, I_V, F_V, T_E, I_E, F_E)$$

is a single-valued neutrosophic directed n-Superhypergraph.

Proof: As above, for each v,

$$T_V(v) + I_V(v) + F_V(v)$$
$$= \max(T_V^1, T_V^2) + \min(I_V^1, I_V^2) + \min(F_V^1, F_V^2)$$
$$\leq T_V^i(v) + I_V^i(v) + F_V^i(v) \leq 3,$$

and for each edge e,

$$T_E(e) \leq \max\{\min_x T_V^1(x), \min_x T_V^2(x)\}$$
$$\leq \min_x \max\{T_V^1(x), T_V^2(x)\} = \min_x T_V(x),$$

with similar arguments for I_E and F_E. Thus the constraints hold. ∎

V. Discussion: Comparative Qualitative Analysis of Graph Concepts

In this section, we briefly present a comparative qualitative analysis of graph concepts in tabular form. We summarize the comparison between fuzzy directed hypergraphs and fuzzy directed n-superhypergraphs in Table VI, and the comparison between neutrosophic directed hypergraphs and neutrosophic directed n-superhypergraphs in Table VII. These comparisons suggest that, relative to the already established class of directed hypergraphs, the directed superhypergraph class introduced in this work is expected to provide a clearer representation of more complex concepts. However, since this paper focuses primarily on theoretical aspects, we believe that quantitative analyses will be necessary in future work.

We present a comparison between neutrosophic n-superhypergraphs and neutrosophic directed n-superhypergraphs in Table VIII. Which of the two models should be used depends on the nature of the phenomenon to be modeled; however, for real-world concepts that are both hierarchical and require directionality, neutrosophic directed n-superhypergraphs are expected to be more suitable.

VI. Conclusion and Future Works

In this paper, we introduced and investigated the concepts of *Fuzzy Directed Superhypergraphs* and *Neutrosophic Directed Superhypergraphs*. These models provide a unified language for hierarchical multiway interactions with orientation under uncertainty. They can represent graded membership and, in the neutrosophic setting, graded truth, indeterminacy, and falsity; thus they support robust inference and decision-making and capture complex causal, layered networks beyond classical graphs and hypergraphs.

As future work, we plan to conduct computational experiments, analyze graph parameters [94], investigate relevant graph classes [95], and study associated graph algorithms. Moreover, we aim to extend these frameworks to *bidirected graphs* [96], [62], [97] and *multidirected graphs* [24], examining their mathematical structures and potential applications. We also intend to explore extensions and normalization methods using advanced concepts such as Neutrosophic OffGraphs, OverGraphs, and UnderGraphs (cf. [62], [98], [99], [100]). Finally, we expect that further quantitative evaluations via computational experiments, as well as the development and assessment of efficient algorithms for these models, will be actively pursued.

Author Contributions

Conceptualization, Takaaki Fujita and Florentin Smarandache; Investigation, Takaaki Fujita; Methodology, Takaaki Fujita; Writing – original draft, Takaaki Fujita; Writing – review & editing, Takaaki Fujita and Florentin Smarandache.

References

[1] R. Diestel, *Graph theory*. Springer (print edition); Reinhard Diestel (eBooks), 2024.

[2] J. L. Gross, J. Yellen, and M. Anderson, *Graph theory and its applications*. Chapman and Hall/CRC, 2018.

[3] C. Berge, *Hypergraphs: combinatorics of finite sets*. Elsevier, 1984, vol. 45.

[4] F. Smarandache, *Extension of HyperGraph to n-SuperHyperGraph and to Plithogenic n-SuperHyperGraph, and Extension of HyperAlgebra to n-ary (Classical-/Neutro-/Anti-) HyperAlgebra*. Infinite Study, 2020.

[5] L. A. Zadeh, "Fuzzy sets," *Information and control*, vol. 8, no. 3, pp. 338–353, 1965.

[6] D. Molodtsov, "Soft set theory-first results," *Computers & mathematics with applications*, vol. 37, no. 4-5, pp. 19–31, 1999.

[7] K. T. Atanassov and K. T. Atanassov, *Intuitionistic fuzzy sets*. Springer, 1999.

[8] F. Smarandache, "A unifying field in logics: Neutrosophic logic." in *Philosophy*. American Research Press, 1999, pp. 1–141.

[9] W.-L. Gau and D. J. Buehrer, "Vague sets," *IEEE transactions on systems, man, and cybernetics*, vol. 23, no. 2, pp. 610–614, 1993.

[10] V. Torra and Y. Narukawa, "On hesitant fuzzy sets and decision," in *2009 IEEE international conference on fuzzy systems*. IEEE, 2009, pp. 1378–1382.

[11] F. Smarandache, "Plithogeny, plithogenic set, logic, probability, and statistics," *arXiv preprint arXiv:1808.03948*, 2018.

[12] Y. Feng, H. You, Z. Zhang, R. Ji, and Y. Gao, "Hypergraph neural networks," in *Proceedings of the AAAI conference on artificial intelligence*, vol. 33, no. 01, 2019, pp. 3558–3565.

[13] T. Fujita and F. Smarandache, "Fundamental computational problems and algorithms for superhypergraphs," *HyperSoft Set Methods in Engineering*, vol. 3, pp. 32–61, 2025.

[14] N. B. Nalawade, M. S. Bapat, S. G. Jakkewad, G. A. Dhanorkar, and D. J. Bhosale, "Structural properties of zero-divisor hypergraph and superhypergraph over $\mathbb{Z}_n$: Girth and helly property," *Panamerican Mathematical Journal*, vol. 35, no. 4S, p. 485, 2025.

[15] M. Hatzel, *Dualities in graphs and digraphs*. Universitätsverlag der Technischen Universität Berlin, 2023.

TABLE VI: Concise comparison between fuzzy directed hypergraphs and fuzzy directed n-superhypergraphs

Aspect	**Fuzzy Directed Hypergraph (FDHG)**	**Fuzzy Directed n-Superhypergraph (FDnSHG)**
Base universe	A vertex set V (typically "atomic" entities).	A base set S with iterated powersets $\mathcal{P}^n(S)$; vertices may be nested objects.
Vertex objects	Vertices are elements $v \in V$.	n-supervertices $v \in V \subseteq \mathcal{P}^n(S)$ (hierarchical / multi-level entities).
Edge objects (direction)	Directed hyperarcs $e = (T(e), H(e))$ with $T(e) \subseteq V$, $H(e) \subseteq V$ (often $H(e) \subseteq V \setminus T(e)$).	Directed n-superedges $e = (\mathrm{Tail}(e), \mathrm{Head}(e)) \in E \subseteq \mathcal{P}^n(S) \times \mathcal{P}^n(S)$, viewed as flow from a set of n-supervertices to another.
Membership on vertices	$\sigma : V \to [0, 1]$.	$\sigma : V \to [0, 1]$ on n-supervertices.
Membership on edges	$\mu : E \to [0, 1]$ on directed hyperarcs.	$\mu : E \to [0, 1]$ on directed n-superedges.
Fuzzy consistency / appurtenance constraint	$\mu(e) \leq \min_{x \in T(e) \cup H(e)} \sigma(x)$.	$\mu(e) \leq \min_{x \in \mathrm{Tail}(e) \cup \mathrm{Head}(e)} \sigma(x)$.
Expressive power (informal)	Captures uncertain multiway *directed* relations among base-level entities.	Captures uncertain multiway *directed* relations among *nested* (multi-level) entities, enabling hierarchical modeling.
Special-case relationship	Base model.	When $n = 0$, $\mathcal{P}^0(S) = S$ and FD0SHG reduces to an FDHG (same type of membership constraint).

[16] D. Berwanger, A. Dawar, P. Hunter, S. Kreutzer, and J. Obdržálek, "The dag-width of directed graphs," *Journal of Combinatorial Theory, Series B*, vol. 102, no. 4, pp. 900–923, 2012.

[17] M. Akram and A. Luqman, "A new decision-making method based on bipolar neutrosophic directed hypergraphs," *Journal of Applied Mathematics and Computing*, vol. 57, pp. 547 – 575, 2017.

[18] T. Fujita and F. Smarandache, *HyperGraph and SuperHyperGraph Theory with Applications*. Neutrosophic Science International Association (NSIA) Publishing House, 2026.

[19] T. Fujita, "Review of some superhypergraph classes: Directed, bidirected, soft, and rough," *Advancing Uncertain Combinatorics through Graphization, Hyperization, and Uncertainization: Fuzzy, Neutrosophic, Soft, Rough, and Beyond (Second Volume)*, 2024.

[20] N. Bowler, E. Ghorbani, F. Gut, R. W. Jacobs, and F. Reich, "Menger's theorem in bidirected graphs," *arXiv preprint arXiv:2306.01921*, 2023.

[21] O. Bessouf, A. Khelladi, and T. Zaslavsky, "Transitive closure and transitive reduction in bidirected graphs," *Czechoslovak Mathematical Journal*, vol. 69, pp. 295 – 315, 2016.

[22] M. DeVos, "Flows on bidirected graphs," *arXiv preprint arXiv:1310.8406*, 2013.

[23] O. Bessouf and A. Khelladi, "New concept of connection in bidirected graphs," *RAIRO Oper. Res.*, vol. 52, p. 351, 2018.

[24] S. Pardo-Guerra, V. K. George, and G. A. Silva, "On the graph isomorphism completeness of directed and multidirected graphs," *Mathematics*, vol. 13, no. 2, p. 228, 2025.

[25] J. Kannan, V. Jayakumar, and M. Pethaperumal, "Advanced fuzzy-based decision-making: the linear diophantine fuzzy codas method for logistic specialist selection," *Spectrum of operational research*, vol. 2, no. 1, pp. 41–60, 2025.

[26] M. Palanikumar and N. Kausar, "Research on the redefined square root interval-valued normal pythagorean fuzzy multi-attribute decision-making model based on aggregation operators," *Spectrum of Operational Research*, pp. 1–18, 2027.

[27] D. Xu, X. Kang, X. Zhang, and M. Zhu, "The best-worst method based on interval neutrosophic sets." *IAENG International Journal of Computer Science*, vol. 51, no. 10, pp. 1527–1533, 2024.

[28] D. Xu, L. Zhou, Y. Zhao, and X. Liu, "Novel three-way decision models based on bayes decision theory with neutrosophic sets." *IAENG International Journal of Applied Mathematics*, vol. 54, no. 5, pp. 910–916, 2024.

[29] B. C. Cuong and V. Kreinovich, "Picture fuzzy sets-a new concept for computational intelligence problems," in *2013 third world congress on information and communication technologies (WICT 2013)*. IEEE, 2013, pp. 1–6.

[30] V. Kamalakannan, P. Murugadas, and M. Kavitha, "T-ordering and minus ordering on picture fuzzy matrices." *IAENG International Journal of Computer Science*, vol. 51, no. 10, pp. 1560–1569, 2024.

[31] P. K. Maji, R. Biswas, and A. R. Roy, "Soft set theory," *Computers & mathematics with applications*, vol. 45, no. 4-5, pp. 555–562, 2003.

[32] F. Smarandache, "Extension of soft set to hypersoft set, and then to plithogenic hypersoft set," *Neutrosophic sets and systems*, vol. 22, no. 1, pp. 168–170, 2018.

[33] Z. Pawlak, "Rough sets," *International journal of computer & information sciences*, vol. 11, pp. 341–356, 1982.

[34] Z. Pawlak, *Rough sets: Theoretical aspects of reasoning about data*. Springer Science & Business Media, 2012, vol. 9.

[35] T. Fujita and F. Smarandache, *A Dynamic Survey of Fuzzy, Intuitionistic Fuzzy, Neutrosophic, , and Extensional Sets*. Neutrosophic Science International Association (NSIA), 2025.

[36] T. Fujita and F. Smarandache, "A unified framework for u-structures and functorial structure: Managing super, hyper, superhyper, tree, and forest uncertain over/under/off models," *Neutrosophic Sets and Systems*, vol. 91, pp. 337–380, 2025.

[37] F. Smarandache, *Plithogenic set, an extension of crisp, fuzzy, intuitionistic fuzzy, and neutrosophic sets-revisited*. Infinite study, 2018.

[38] A. Rosenfeld, "Fuzzy graphs," in *Fuzzy sets and their applications to cognitive and decision processes*. Elsevier, 1975, pp. 77–95.

[39] H. Rashmanlou, S. Samanta, M. Pal, and R. A. Borzooei, "Intuitionistic fuzzy graphs with categorical properties," *Fuzzy information and Engineering*, vol. 7, no. 3, pp. 317–334, 2015.

[40] M. Akram, "Bipolar fuzzy graphs," *Information sciences*, vol. 181, no. 24, pp. 5548–5564, 2011.

[41] K. Luo and W. Gao, "Independent set in bipolar fuzzy graph." *IAENG International Journal of Applied Mathematics*, vol. 52, no. 4, pp. 846–854, 2022.

[42] J. Lu, L. Zhu, and W. Gao, "Bipolar fuzzy influence graph and its chemical application." *Engineering Letters*, vol. 31, no. 4, pp. 1508–1517, 2023.

[43] L. Lin, M. Zhang, and L. Ma, "Solving the telecommunication network problem using vague graph." *Engineering Letters*, vol. 28, no. 4, pp. 1213–1219, 2020.

[44] S. Broumi, M. Talea, A. Bakali, and F. Smarandache, "Single valued neutrosophic graphs," *Journal of New theory*, vol. 10, pp. 86–101, 2016.

[45] S. Broumi, M. Talea, A. Bakali, and F. Smarandache, "Interval valued neutrosophic graphs," *Critical Review, XII*, vol. 2016, pp. 5–33, 2016.

[46] T. Fujita, "A short note on the basic graph construction algorithm for plithogenic graphs," *Advancing Uncertain Combinatorics through Graphization, Hyperization, and Uncertainization: Fuzzy, Neutrosophic, Soft, Rough, and Beyond*, p. 274, 2025.

[47] F. Sultana, M. Gulistan, M. Ali, N. Yaqoob, M. Khan, T. Rashid, and T. Ahmed, "A study of plithogenic graphs: applications in spreading coronavirus disease (covid-19) globally," *Journal of ambient intelligence and humanized computing*, vol. 14, no. 10, pp. 13 139–13 159, 2023.

[48] R. J. Hussain and M. S. A. Farhana, "Fuzzy chromatic number of fuzzy soft cycle and complete fuzzy soft graphs," *AIP Conference Proceedings*, 2023.

TABLE VII: Concise comparison between neutrosophic directed hypergraphs and neutrosophic directed n-superhypergraphs

Aspect	**Neutrosophic Directed Hypergraph (NDHG)**	**Neutrosophic Directed n-Superhypergraph (NDnSHG)**
Base universe	A nonempty ground set X (vertices).	A nonempty base set S with iterated powersets $\mathcal{P}^n(S)$.
Vertex objects	Vertices are elements of X equipped with neutrosophic degrees.	n-supervertices $v \in V \subseteq \mathcal{P}^n(S)$, i.e., nested / multi-level entities.
Directed edge objects	Neutrosophic directed hyperarcs $e = (T(e), H(e))$ with $T(e), H(e) \subseteq X$ (tail/head).	Directed n-superedges $e = (\mathrm{Tail}(e), \mathrm{Head}(e)) \in E \subseteq \mathcal{P}^n(S) \times \mathcal{P}^n(S)$.
Membership structure	Typically three degrees for vertices and (depending on the model) three degrees for each hyperarc, encoding truth/indeterminacy/falsity.	Three degrees for n-supervertices and three degrees for directed n-superedges: $T_V, I_V, F_V : V \to [0,1]$ and $T_E, I_E, F_E : E \to [0,1]$.
Constraints (typical)	Vertex constraint: $0 \leq T(v) + I(v) + F(v) \leq 3$. Edge constraint: edge-degrees are bounded by vertex-degrees on incident endpoints (model-dependent).	Vertex constraint: $0 \leq T_V(v)+I_V(v)+F_V(v) \leq 3$. Edge-appurtenance constraints: $T_E(e) \leq \min_{x \in \mathrm{Tail}(e) \cup \mathrm{Head}(e)} T_V(x)$, $I_E(e) \leq \min_{x \in \mathrm{Tail}(e) \cup \mathrm{Head}(e)} I_V(x)$, $F_E(e) \leq \min_{x \in \mathrm{Tail}(e) \cup \mathrm{Head}(e)} F_V(x)$.
Expressive power (informal)	Uncertain multiway *directed* relations among base-level vertices.	Uncertain multiway *directed* relations among *hierarchical* (nested) vertex-objects, enabling multi-layer modeling.
Special-case relationship	Base model.	When $n = 0$, $\mathcal{P}^0(S) = S$, and ND0SHG reduces to a neutrosophic directed hypergraph on S under matching conventions.

[49] M. Saeed, A. U. Rahman, and M. Arshad, "A study on some operations and products of neutrosophic hypersoft graphs," *Journal of Applied Mathematics and Computing*, vol. 68, no. 4, pp. 2187–2214, 2022.

[50] R. Noor, I. Irshad, and I. Javaid, "Soft rough graphs," *arXiv: General Mathematics*, 2017.

[51] M. Farshi and B. Davvaz, "Generalized fuzzy hypergraphs and hypergroupoids," *Filomat*, vol. 30, no. 9, pp. 2375–2387, 2016.

[52] J. N. Mordeson and P. S. Nair, *Fuzzy graphs and fuzzy hypergraphs*. Physica, 2012, vol. 46.

[53] M. Akram and A. Luqman, "Intuitionistic single-valued neutrosophic hypergraphs," *Opsearch*, vol. 54, pp. 799–815, 2017.

[54] M. Akram, S. Shahzadi, and A. B. Saeid, "Single-valued neutrosophic hypergraphs," *viXra*, pp. 1–14, 2018.

[55] F. Smarandache, *Introduction to the n-SuperHyperGraph-the most general form of graph today*. Infinite Study, 2022.

[56] M. Hamidi and M. Taghinezhad, *Application of Superhypergraphs-Based Domination Number in Real World*. Infinite Study, 2023.

[57] T. Fujita, "Modeling hierarchical systems in graph signal processing, electric circuits, and bond graphs via hypergraphs and superhypergraphs," *Journal of Engineering Research and Reports*, vol. 27, no. 5, p. 542, 2025.

[58] M. Hamidi, F. Smarandache, and M. Taghinezhad, *Decision Making Based on Valued Fuzzy Superhypergraphs*. Infinite Study, 2023.

[59] T. Batool and U. Ahmad, "A new approximation technique based on central bodies of rough fuzzy directed graphs for agricultural development," *Journal of Applied Mathematics and Computing*, vol. 70, no. 2, pp. 1673–1705, 2024.

[60] E. L. Enriquez, G. M. Estrada, C. Loquias, R. J. Bacalso, and L. A. Ocampo, "Domination in fuzzy directed graphs," *Mathematics*, 2021.

[61] T. Fujita and F. Smarandache, "Mixed graph in fuzzy, neutrosophic, and plithogenic graphs," *Neutrosophic Sets and Systems*, vol. 74, pp. 457–479, 2024.

[62] T. Fujita, "Review of plithogenic directed, mixed, bidirected, and pangene offgraph," *Advancing Uncertain Combinatorics through Graphization, Hyperization, and Uncertainization: Fuzzy, Neutrosophic, Soft, Rough, and Beyond*, p. 120, 2024.

[63] A. Luqman, M. Akram, and B. Davvaz, "q-rung orthopair fuzzy directed hypergraphs: A new model with applications," *J. Intell. Fuzzy Syst.*, vol. 37, pp. 3777–3794, 2019.

[64] W. A. Khan, W. Arif, Q. H. Nguyen, T. T. Le, and H. Van Pham, "Picture fuzzy directed hypergraphs with applications towards decision-making and managing hazardous chemicals," *IEEE Access*, 2024.

[65] M. Akram, A. Luqman, M. Akram, and A. Luqman, "Extended bipolar fuzzy (directed) hypergraphs to m-polar information," *Fuzzy Hypergraphs and Related Extensions*, pp. 187–234, 2020.

[66] M. Akram and A. Luqman, *Fuzzy hypergraphs and related extensions*. Springer, 2020.

[67] M. Akram and A. Luqman, "Certain networks models using single-valued neutrosophic directed hypergraphs," *Journal of Intelligent & Fuzzy Systems*, vol. 33, no. 1, pp. 575–588, 2017.

[68] P. Dhanya and P. Ramkumar, "Text analysis using morphological operations on a neutrosophic text hypergraph," *Neutrosophic Sets and Systems*, vol. 61, pp. 337–364, 2023.

[69] M. Hamidi and F. Smarandache, "Single-valued neutrosophic directed (hyper) graphs and applications in networks," *Journal of Intelligent & Fuzzy Systems*, vol. 37, no. 2, pp. 2869–2885, 2019.

[70] M. Akram, A. N. Gani, and A. B. Saeid, "Vague hypergraphs," *Journal of Intelligent & Fuzzy Systems*, vol. 26, no. 2, pp. 647–653, 2014.

[71] H. Sahakyan, "On the set of simple hypergraph degree sequences," *Applied Mathematical Sciences*, vol. 9, no. 5, pp. 243–253, 2015.

[72] Y. Gao, Z. Zhang, H. Lin, X. Zhao, S. Du, and C. Zou, "Hypergraph learning: Methods and practices," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 44, no. 5, pp. 2548–2566, 2020.

[73] E. M. C. Valencia, J. P. C. Vásquez, and F. Á. B. Lois, "Multineutrosophic analysis of the relationship between survival and business growth in the manufacturing sector of azuay province, 2020–2023, using plithogenic n-superhypergraphs," *Neutrosophic Sets and Systems*, vol. 84, pp. 341–355, 2025.

[74] J. C. M. Bravo, C. J. B. Piedrahita, M. A. M. Bravo, and L. M. Pilacuan-Bonete, "Integrating smed and industry 4.0 to optimize processes with plithogenic n-superhypergraphs," *Neutrosophic Sets and Systems*, vol. 84, pp. 328–340, 2025.

[75] B. V. S. Marcos, M. F. Willner, B. V. C. Rosa, F. F. R. M. Yissel, E. R. Roberto, L. D. B. Puma, and D. M. M. Fernández, "Using plithogenic n-superhypergraphs to assess the degree of relationship between information skills and digital competencies," *Neutrosophic Sets and Systems*, vol. 84, pp. 513–524, 2025.

[76] S. Zhu, "Neutrosophic n-superhypernetwork: A new approach for evaluating short video communication effectiveness in media con-

TABLE VIII: Concise comparison between neutrosophic n-superhypergraphs and neutrosophic directed n-superhypergraphs

Aspect	**Neutrosophic n-Superhypergraph (NnSHG)**	**Neutrosophic Directed n-Superhypergraph (NDnSHG)**
Base universe	Base set S with $V \subseteq \mathcal{P}^n(S)$, $E \subseteq \mathcal{P}^n(S)$ (undirected superedges).	Base set S with $V \subseteq \mathcal{P}^n(S)$, $E \subseteq \mathcal{P}^n(S) \times \mathcal{P}^n(S)$ (directed superedges).
Edge objects	Undirected n-superedges are subsets $e \subseteq V$.	Directed n-superedges are ordered pairs $e = (\mathrm{Tail}(e), \mathrm{Head}(e))$.
Directionality	No intrinsic direction; incidence is symmetric.	Encodes asymmetric "flow" from $\mathrm{Tail}(e)$ to $\mathrm{Head}(e)$.
Membership structure (typical)	Vertex degrees $T_V, I_V, F_V : V \to [0,1]$. Edge degrees assigned to each superedge (either as $T_E, I_E, F_E : E \to [0,1]$ or per-incidence (e,v), depending on the chosen convention).	Vertex degrees $T_V, I_V, F_V : V \to [0,1]$ and directed-edge degrees $T_E, I_E, F_E : E \to [0,1]$.
Appurtenance constraints (canonical form)	Edge-degrees are bounded by the vertex-degrees of incident $v \in e$, e.g. $T_E(e) \leq \min_{v \in e} T_V(v)$ (and similarly for I, F), under the edge-level convention.	Directed version bounds edge-degrees by all endpoints in $\mathrm{Tail}(e) \cup \mathrm{Head}(e)$, i.e. $T_E(e) \leq \min_{x \in \mathrm{Tail}(e) \cup \mathrm{Head}(e)} T_V(x)$ (and similarly for I, F).
Expressive power (informal)	Hierarchical uncertainty without direction; suitable for symmetric multi-level relations.	Hierarchical uncertainty *with direction*; suitable for causal/precedence/influence-type relations across levels.
Embedding	Base model.	Any NnSHG can be embedded into NDnSHG by replacing each undirected superedge e with a directed superedge (e, e) (and keeping the same degrees under consistent conventions).

vergence," *Neutrosophic Sets and Systems*, vol. 85, pp. 1004–1017, 2025.

[77] A. Bretto, "Hypergraph theory," *An introduction. Mathematical Engineering. Cham: Springer*, vol. 1, 2013.

[78] F. Smarandache, "Foundation of superhyperstructure & neutrosophic superhyperstructure," *Neutrosophic Sets and Systems*, vol. 63, no. 1, p. 21, 2024.

[79] F. Smarandache, "The cardinal of the m-powerset of a set of n elements used in the superhyperstructures and neutrosophic superhyperstructures," *Systems Assessment and Engineering Management*, vol. 2, pp. 19–22, 2024.

[80] M. Jdid, F. Smarandache, and T. Fujita, "A linear mathematical model of the vocational training problem in a company using neutrosophic logic, hyperfunctions, and superhyperfunction," *Neutrosophic Sets and Systems*, vol. 87, pp. 1–11, 2025.

[81] F. Smarandache, "Introduction to superhyperalgebra and neutrosophic superhyperalgebra," *Journal of Algebraic Hyperstructures and Logical Algebras*, 2022.

[82] F. Smarandache, "n-superhypergraph and plithogenic n-superhypergraph," *Nidus Idearum*, vol. 7, pp. 107–113, 2019.

[83] L. A. Zadeh, "Fuzzy logic, neural networks, and soft computing," in *Fuzzy sets, fuzzy logic, and fuzzy systems: selected papers by Lotfi A Zadeh*. World Scientific, 1996, pp. 775–782.

[84] L. A. Zadeh, "A note on z-numbers," *Information sciences*, vol. 181, no. 14, pp. 2923–2932, 2011.

[85] T. Nishad, T. A. Al-Hawary, and B. M. Harif, "General fuzzy graphs," *Ratio Mathematica*, vol. 47, 2023.

[86] S. Samanta and M. Pal, "Bipolar fuzzy hypergraphs," *International Journal of Fuzzy Logic Systems*, vol. 2, no. 1, pp. 17–28, 2012.

[87] J. Ding, W. Bai, and C. Zhang, "A new multi-attribute decision making method with single-valued neutrosophic graphs," *International Journal of Neutrosophic Science*, 2021.

[88] M. Hamidi and A. B. Saeid, "Accessible single-valued neutrosophic graphs," *Journal of Applied Mathematics and Computing*, vol. 57, pp. 121–146, 2018.

[89] S. S. Hussain, N. Durga, R. Hossein, and G. Ganesh, "New concepts on quadripartitioned single-valued neutrosophic graph with real-life application," *International Journal of Fuzzy Systems*, vol. 24, no. 3, pp. 1515–1529, 2022.

[90] M. Akram and H. S. Nawaz, "Implementation of single-valued neutrosophic soft hypergraphs on human nervous system," *Artificial Intelligence Review*, vol. 56, no. 2, pp. 1387–1425, 2023.

[91] M. Akram and H. S. Nawaz, "Algorithms for the computation of regular single-valued neutrosophic soft hypergraphs applied to supranational asian bodies," *Journal of Applied Mathematics and Computing*, vol. 68, no. 6, pp. 4479–4506, 2022.

[92] M. Wientjes, "Hyperlink prediction with modular directed hypergraph neural networks," Ph.D. dissertation, Master's thesis, Eindhoven University of Technology, 2021.

[93] Z. Ma, W. Zhao, and Z. Yang, "Directed hypergraph representation learning for link prediction," in *International conference on artificial intelligence and statistics*. PMLR, 2024, pp. 3268–3276.

[94] D. J. Harvey and D. R. Wood, "Parameters tied to treewidth," *Journal of Graph Theory*, vol. 84, no. 4, pp. 364–385, 2017.

[95] A. Brandstädt, V. B. Le, and J. P. Spinrad, *Graph classes: a survey*. SIAM, 1999.

[96] N. Kita, "Bidirected graphs i: Signed general kotzig-lovász decomposition," *arXiv: Combinatorics*, 2017.

[97] J. A. J. González and A. Mróz, "Bidirected graphs, integral quadratic forms and some diophantine equations," *Discrete Mathematics*, vol. 349, no. 3, p. 114811, 2026.

[98] N. E. Arif *et al.*, "Domination (set and number) in neutrosophic soft over graphs," *Wasit Journal for Pure sciences*, vol. 1, no. 3, pp. 26–43, 2022.

[99] A. S. majeed and N. E. Arif, "Domination (set and number) in neutrosophic soft over graphs," *Wasit Journal of Pure sciences*, 2022.

[100] R. Devi, "Minimal domination via neutrosophic over graphs," *1ST INTERNATIONAL CONFERENCE ON MATHEMATICAL TECHNIQUES AND APPLICATIONS: ICMTA2020*, 2020.

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